Control of Shock Positions with Applications to Sonic Booms

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There are many fluid flow problems with discontinuities in the data or in the flow. Among them two are quite important for applications:

- transonic and supersonic flow with shocks and buffeting
- acoustics with sonic boom

Optimisation of these systems by standard gradient methods requires the application of the techniques of the Calculus of Variations and an implicit assumption that a Taylor expansion exists with respect to the degrees of freedom of the problem. Take for example the flow in a transonic nozzle and the variation of the flow with respect to the inflow conditions; when these vary the shock moves and the derivative of the flow variables with respect to inflow conditions is a Dirac measure and so the Taylor expansion does not exists.

By extending the calculus of variation via the theory of distribution it is possible to show however that the derivatives exist. But the result has serious numerical implications; in particular it favors the mixed finite element methods.

We shall give numerical illustrations using the finite element method for an inverse problem for Burger's equation, for the design of a transonic nozzle and for the design of a business supersonic airplane for sonic boom minimization.