Linearly additive random processes with independent increments on any time-like curve

Shigeo TAKENAKA
Dept.of Appl. Math. Okayama University of Science

March, 2003

Definition 1 (linearly additive processes) *An $\mathbf{R}^n$-parameter stochastic process $\{X(\mathbf{t}); t \in \mathbf{R}^n\}$ is called a* <u>*linearly additive process*</u> *if it is additive on any lines. That is, the process $Z(s) \equiv X(s\mathbf{v} + \mathbf{v_0})$ has independent increments for any line $\{s\mathbf{v} + \mathbf{v_0}\}$.*

T.Mori obtained the following theorem:

Theorem 2 T.Mori[92] *Let $\{X(\mathbf{t})\}$ be an $\mathbf{R}^n$-parameter linearly additive process. Then, there uniquely exists a measure μ on the set of all hyper-planes (of co-dimension 1) in $\mathbf{R}^n$ and the process has the representation*

$$X(\mathbf{t}) = Y(S(\mathbf{t})),$$

where $S(\mathbf{t})$ is the connected component of $\mathbf{R}^n \setminus \mathbf{t}^*$ which does not include the origin and $\{Y(B); B$ measurable in $\mathbf{R}^n\}$ is the $S\alpha S$-random measure with control measure $(\mathbf{R}^n, \mu)$. Here we identify the space of hyperplanes with $\mathbf{R}^n$ by the projective duality map *.

Let us call this measure the Chentsov-Mori mesure of $\{X\}$.

Definition 3 (multi-parameter additive process, K.Sato[00]**)** *An n-parameter stochastic process $\{X(\mathbf{t}); t \in \mathbf{R}^n\}$ is called a* <u>*multi-parameter*</u> <u>*additive process*</u> *if:*

1. *For any points, $\mathbf{s}_1 \preceq \mathbf{s}_2 \preceq \cdots \preceq \mathbf{s}_m, \cdots, X(\mathbf{s}_n) - X(\mathbf{s}_{n-1})$ makes an independent system, ,where $\mathbf{u} = (u_1.u_2, \cdots, u_n) \preceq \mathbf{t} = (t_1, t_2, \cdots, t_n)$ that is $u_1 \leq t_1, u_2 \leq t_2, \cdots, u_n \leq t_n$, $X(\mathbf{s}_2) - X(\mathbf{s}_1), X(\mathbf{s}_3) - X(\mathbf{s}_2)$.*

2. *If $\mathbf{s}_1 \preceq \mathbf{s}_2$, $\mathbf{s}_3 \preceq \mathbf{s}_4$ and $\mathbf{s}_2 - \mathbf{s}_1 = \mathbf{s}_4 - \mathbf{s}_3$, $X(\mathbf{s}_2) - X(\mathbf{s}_1)$ and $X(\mathbf{s}_4) - X(\mathbf{s}_3)$ are subject to the same law (i.e. $\{X\}$ is invariant under the parallel transforms).*

3. *$X(\mathbf{0}) = 0$, a.s.*

4. *$X(\mathbf{s})$ is right continuous and has left limits with respect to the order $\preceq$.*

Theorem 4 *Let $\{X(\mathbf{t}); \mathbf{t} \in \mathbf{R}^n\}$ be a linearly additive multi-parameter prcess. Then there uniquely exists a measure Φ which concentrates on $S^{n-1} \cap \mathbf{R}^n_-$, the process $\{X\}$ has the following Chentsov type representation:*

$$X(\mathbf{t}) = Y(S(\mathbf{t})).$$

Where $\mathbf{Y} = \{Y\}$ is the random mesure controled by the mesure Φ which is invariant under the dual action of the Euclidean motion group.

Let $\mathbf{V} \in \mathbf{R}_+{}^n$ be a convex cone. A curve $L = \{\ell(t); t \in \mathbf{R}_+\} \in \mathbf{V}$ is called a time like curve if $\{\ell(s); s \geq t\} \in \ell(t) + \mathbf{V}$.

Definition 5 *A process $\{X(\mathbf{t}); \mathbf{t} \in V\}$ is called an additive process with respect to $\mathbf{V}$ if the parameter restriction $\{X_L(t) = X(\ell(t)\}$ is additive for any time-like curve L.*

Note that $\mathbf{R}_+{}^n$ is a convex cone. Thus the above multi-parameter additive process is an exmple of addive process with respect to the cone $\mathbf{R}_+{}^n$.

Theorem 6 *Let $\{X(\mathbf{t}); \mathbf{t} \in \mathbf{R}^n\}$ be a linearly additive process and $\mathbf{V}$ be a convex cone. If $\{X(\mathbf{t}; \mathbf{t} \in \mathbf{V}\}$ is an additive process with respect to $\mathbf{V}$, then there uniquely exists a measure Φ which concentrates in the dual conve $\mathbf{V}^*$ and the process $\{X\}$ has the following Chentsov type representation:*

$$X(\mathbf{t}) = Y(S(\mathbf{t})).$$

Recently, my graduate student, Miss H.Tanida has successed to characterize the sub-ordinators valued on time-like curved. So, now we are ready to consider **a multi-parameter version of sub-ordination.**

References

[Mori92] Mori, T, *Representation of linearly additive random fields.* Prob. Theory and Related Fields **92** 91–115,(1992)

[KSato00] Sato, K, On Multivariate subordination and selfdecomposability. abstract of a workshop at Inst. Math. Stat. Oct. 2000

[HTanida03] anida, H. *On Cone-valued Subordinators* Master Theses, Okayama University of Science, Feb. 2003

[STakenaka03] akenaka, S. *Linearly additive random fields with independent increments on time-like curves Probabilty and Mathematical Statistics, accepted (2003)*