Determinsms of Set-Indexed stable processes

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Let $\mathcal{Y} = \{Y(B); B \in \mathcal{B}\}$ be the symmetric α stable random mesure controled by a measure space $(E, \mathcal{B}, \mu)$, $0 < \alpha \leq 2$. That is,

1. $\mathbf{E}[e^{izY(B)}] = e^{-\mu(B)|z|^\alpha}$,

2. $Y(B_1)$ and $Y(B_2)$ is independent if $B_1 \cap B_2 = \emptyset$.

3. $Y(A) = \sum Y(B_n)$, a.s. if $A = \bigcup B_n$, $B_i \cap B_j = \emptyset$.

A stochastic process, $\mathcal{X} = \{X(t),\ t \in \mathbf{T}\}$ is called a set-indexed process if there exist a random mesure $\mathcal{Y}$ above and a map $S. : \mathbf{T} \to \mathcal{B}$ and it holds $X(t) = Y(S_t)$.

For example, let take $(E, \mathcal{B}, \mu)$ be the d-dimensional Euclidean space with Lebesgue measure and set $S_{\mathbf{t}} = \{|\mathbf{x} - \mathbf{t}| < 1; x \in \mathbf{R}^d\}$, $\mathbf{t} \in \mathbf{R}$. Then $\mathcal{X} = \{X(\mathbf{t}) = Y(S_{\mathbf{t}})$ is a stationary stable process with parameter in $\mathbf{R}^{\mathbf{d}}$.

There are several important processes which are expressed as set indexed processes.

1. The multi-parameter Lévy motions—the stable version of multi-parameter Brownian motion.

2. Some class of self-similar processes.

3. Multi-parameter additive processes. These processes are the subjects of my second talk. The parameter restriction of the process on any time-like curve becomes an additive process.

Let return the above example. It is well known that in $\mathbf{R}^2$, 3-circles can divide the plain into 2^3 parts but 4-circles can divide at most 14 parts (see figures).

From this elementary fact it follows that;

any 4-dimensional marginal distribution of the process can be calcultes by their own 3-dimensional marginals.

In general in d-dimensional Euclidean space, there exist configurations of $d+1$ spheres which devide the whole space into 2^{d+1} sub-regions but no configurations of $d+2$ spheres which devide the space into 2^{d+2} regions.

This fact tells us that

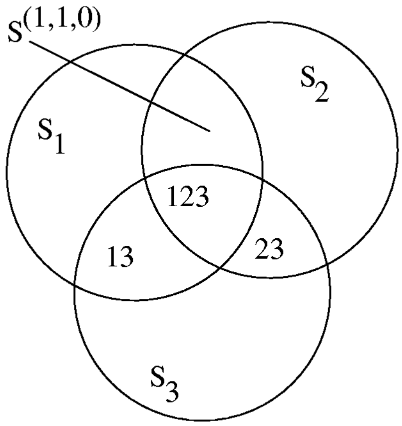

figure 1.

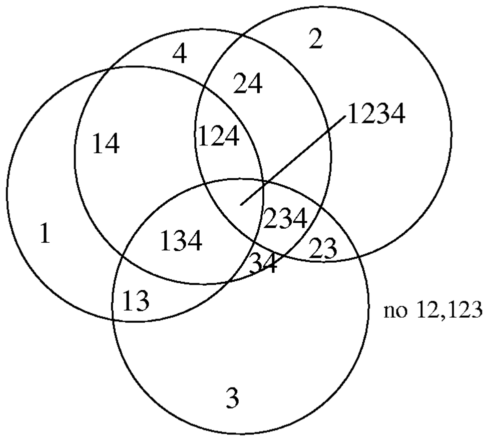

figure 2.

the set indexed process we are considering now is determined by $d+1$-dimensional marginal distributions.

Similar determinsms occures as follows;

1. $\mathbf{R^d}$ parameter Lévy motions: d-dimensional determinisms.

2. self-similar processes: d+2-dimensional or higer-dimensional determinisms.

3. Multi-parameter additive processes: d-dimensional determinsms.

References

[1] Sato, Y. *Distributions of stable random fields of Chentsov type.* Nagoya Math. J. **123,** pp. 119–139 (1991)

[2] Sato, Y. and Takenaka, S. *On determinism of symmetric α-stable processes of generalized Chentsov type.* 'Gaussian Random Fields', pp. 332–345, World Scientific (1991)

[3] Takenaka, S. *Integral-geometirc constructions of self-similar stable processes.* Nagoya Math. J. **123,** pp. 1–12 (1991)

[4] Takenaka, S. *Examples of self-similar stable processes.* 'Stochastic Processes', pp. 303-311, Springer-Verlag (1993)

[5] Takenaka, S. *Linearly additive random fields with independent increments on time-like curves* Probabilty and Mathematical Statistics, accepted (2003)

[6] Mori, T. *Representation of linearly additive random fields.* Prob. Theory and Related Fields **92,** pp. 91–115 (1992)