Title: The Near-critical random graph: structure, diameter and mixing time

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Abstract: In this talk, I will present a complete description of the giant component of the Erdős-Rényi random graph $G(n,p)$ as soon as it emerges from the scaling window, i.e., for $p = (1+\varepsilon)/n$ where $\varepsilon^3 n \to \infty$ and $\varepsilon = o(1)$.

Our description is particularly simple for $\varepsilon = o(n^{-1/4})$, where the giant component C_1 is contiguous with the following model (i.e., every graph property that holds with high probability for this model also holds w.h.p. for C_1). Let Z be normal with mean $\frac{2}{3}\varepsilon^3 n$ and variance $\varepsilon^3 n$, and let K be a random 3-regular graph on $2\lfloor Z \rfloor$ vertices. Replace each edge of K by a path, where the path lengths are i.i.d. geometric with mean $1/\varepsilon$. Finally, attach an independent Poisson$(1-\varepsilon)$-Galton-Watson tree to each vertex.

A similar picture is obtained for larger $\varepsilon = o(1)$, in which case the random 3-regular graph is replaced by a random graph with N_k vertices of degree k for $k \geq 3$, where N_k has mean and variance of order $\varepsilon^k n$.

Based on this description, we show that for any $\varepsilon = o(1)$ with $\varepsilon^3 n \to \infty$, the diameter of C_1 is w.h.p. asymptotic to $D(\varepsilon, n) = (3/\varepsilon)\log(\varepsilon^3 n)$. Furthermore, we prove that the mixing time for the random walk on C_1 is w.h.p. of order $\varepsilon^{-3}\log^2(\varepsilon^3 n)$.

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