EXERCISE FROM PROF. MAGIDOR'S LECTURES IN AII SUMMER SCHOOL

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1. LECTURE 1

Exercise 1. *Suppose f,g are $<_I$ exact upper bounds for $\mathcal{F} \subseteq Ord^A$, show that $f =_I g$.*

Exercise 2. *Find an example of $\langle g_\alpha | \alpha < \omega_1 \rangle \subseteq \omega^\omega$, such that it is $<_{[\omega]^{<\omega}}$-increasing but has no e.u.b.*

Exercise 3. *Suppose $\langle g_\alpha | \alpha < \lambda \rangle$ is strongly $<_I$-increasing in Ord^A/I, where $\lambda > |A|$ is a regular cardinal. Show that $\langle g_\alpha | \alpha < \lambda \rangle$ has an $<_I$ exact upper bound.*

Exercise 4. *Suppose $\langle g_\alpha | \alpha < \lambda \rangle$ is $<_I$-increasing with h an $<_I$ exact upper bound. Suppose it satisfies $(*)_\kappa$, where $|A| < \kappa < \lambda$ and both κ, λ are regular cardinals. Show that*

$$\{a | cf(h(a)) < \kappa\} \in I$$

Exercise 5. *Suppose λ is regular and $\lambda > 2^{|A|}$,show that $\langle g_\alpha | \alpha < \lambda \rangle$ satisfies $(*)_{|A|^+}$.*

Hint: Use Erős-Rado Theorem:

$$(2^\kappa)^+ \to (\kappa)^2_{\kappa^+}$$

Exercise 6. *Suppose $\kappa > \aleph_2$ is a regular cardinals. $S \subseteq S^\kappa_\omega$ is a stationary set. Show that there is a club-guessing sequence $\langle C_\alpha | \alpha \in S \rangle$ for κ.*

2. LECTURE 2

Exercise 7. *Show that $\prod\{\aleph_n | n \in \omega\}/[\omega]^{<\omega}$ is $\aleph^+_\omega$-directed.*

Exercise 8. *Let I be an ideal on A, $h \in Ord^A/I$, $B = \{cf(h(a)) | a \in A\}$ be such that $|A| < min(B)$. Let $J = \{X \subset B | \{a | cf(h(a)) \in X\} \in I\}$.Show that:*

- *J is an ideal on B.*
- *For each $a \in A$,let $\{\zeta^a_\alpha | \alpha \in cf(h(a))\}$ be a strictly increasing sequence in $h(a)$. For each $f \in \prod B$,define $\overline{f} \in \prod A$:*

$$\overline{f}(a) = \zeta^a_{f(cf(h(a)))}$$

Show that: if $f =_J g$,then $\overline{f} =_I \overline{g}$; and if $f \leq_J g$,then $\overline{f} \leq_I \overline{g}$.

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3. LECTURE 3

Exercise 9. *If κ carries a Jonsson algebra, then so is κ^+.*

Exercise 10. *If κ is a regular cardinal with a non-reflecting stationary subset, then κ carries a Jonsson algebra.*

Exercise 11. *If κ is a regular cardinal,then κ^+ carries a Jonsson algebra.*

Definition 3.1. A cardinal is Jonsson cardinal if it does not carry a Jonsson algebra.

Exercise 12. *If κ is measurable, then it is Jonsson.*

Exercise 13. *If κ is measurable and $\mathcal{P}_\kappa$ is the basic Prikry forcing at κ, then $\mathcal{V}^{\mathcal{P}_\kappa} \models \kappa$ is Jonsson.*

4. LECTURE 4

Exercise 14. *Let A be progressive set of regular cardinals, $B \subseteq pcf(A)$ is progressive, then $pcf(B) \subseteq pcf(A)$.*

5. LECTURE 5

Exercise 15. *Show that $cov(\kappa^{++}, \kappa) = \kappa^{++}$.*

Exercise 16. *Let $\mathcal{F}^* = \{M \cap \mu | M \prec H_\theta, M \text{ is } \omega \text{ closed } \}$. Show that $\mathcal{F}^*$ is a covering family.*

Exercise 17. *If $B \subseteq H_\theta$, $|B| = \kappa$, then $\exists M \prec H_\theta$ such that $B \subseteq M$ and M is κ-presentable.*

Exercise 18. *If M is κ-presentable,$A \in M$ and $|A| \leq \kappa$,then $A \subseteq M$.*

Exercise 19. *M is κ-presentable, $\omega < \kappa$, show that M is ω closed.*

6. LECTURE 6

Exercise 20. *Let $\aleph_\delta$ be a singular cardinal such that $\delta < \aleph_\delta$, then $cov(\aleph_\delta, |\delta|) < \aleph_{|\delta|^{+4}}$.*

If the exercise above are too easy for you, how about the following:

Exercise 21. *pcf-conjecture: If A is a progressive set of regular cardinals, then $|pcf(A)| = |A|$.*

Exercise 22. *Improve Shelah's theorem on the bound of $2^{\aleph_\omega}$.*