Let $M = \mathbf{H}^3/\Gamma$ be an orientable hyperbolic 3-manifold. A *Margulis number* for M is a positive real number μ with the following property: for any $P \in \mathbf{H}^3$, if x and y are elements of Γ such that $\max(d(P, x \cdot P), d(P, y \cdot P)) < \mu$, then x and y commute. Culler and I have recently shown that if M is homeomorphic to the interior of a Haken manifold N then 0.286 is a Margulis number for M, and if in addition N has non-empty boundary then 0.292 is a Margulis number for M. I have combined this with character-variety techniques to show that all but at most finitely many closed, orientable hyperbolic 3-manifolds admit 0.292 as a Margulis number. Using different arguments, I have shown that if $0 < \lambda < (\log 3)/2$, there is an explicit bound on the rank of $\pi_1(M)$ for all closed, orientable hyperbolic 3-manifolds M that do not admit λ as a Margulis number. The proof actually gives a bound on the minimal index of a rank-2 subgroup of $\pi_1(M)$. Combining the proof of this last result with earlier work of mine, I can give an explicit bound, in terms of the degree of a number field K, on the number of $\mathbf{Z}_6$-homology 3-spheres with trace field K that do not admit 0.183 as a Margulis number.