AdS geometry as a tool for Teichmüller theory

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IMS
July 29, 2010

Goals of this talk

- Advertise 3d AdS geometry as a tool for Teichmüller theory,
- Explain basics of AdS geometry,
- State some recent results obtained using AdS,
- Examples of proofs,
- Some open questions here and there.

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Recall that

$$H^3 = \{x \in \mathbb{R}^{3,1} \mid \langle x, x \rangle = -1 \& x_0 > 0\} .$$

$$AdS_3 = \{x \in \mathbb{R}^{2,2} \mid \langle x, x \rangle = -1\} .$$

Lorentz analog of H^3 : complete, constant curvature -1.
From relativity : "Anti de Sitter", model for gravity (no matter).
Lorentz analog of S^3 : $PSL(2, \mathbb{R})$ w/ Killing metric, isometry group, etc
Basic idea : hyperbolic and AdS 3-mflds as tools for Teichmüller theory.

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Hyperbolic 3-manifolds and Teichmüller theory

Based (mostly) on quasifuchsian 3-manifolds. Examples of applications include :

- complex projective structures on surfaces,
- complex earthquakes (McMullen),
- the volume of the convex core of quasifuchsian manifolds is coarsely equivalent to the Weil-Petersson distance between the metrics on its boundary (Brock),
- the renormalized volume as a Kähler potential for WP,
- properties of the grafting map.

Not developed here.

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AdS 3-manifolds and Teichmüller theory

Some aspects :

- earthquakes,
- extensions of the earthquake flow,
- minimal Lagrangian diffeos.

AdS side involves physically relevant notions :

- globally hyperbolic (GH) spaces (analogs of quasifuchsian),
- "particles",
- multi-black holes,
- maximal surfaces.

Notations : S closed surface of genus ≥ 2, $\mathcal{T}$ Teichmüller space.

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Measured laminations

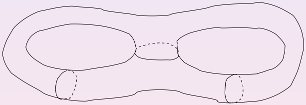
$\mathcal{WM}$ = { weighted multicurves on S } : set of disjoint simple closed curves, each with a positive weight.

$\mathcal{WM}$ is infinite : simple closed curves on S can wrap around a lot.

Let $(c_i, l_i)_{i=1,\cdots,n} \in \mathcal{WM}$, the c_i form a *lamination* and the l_i define a *transverse measure* : gives a total weight to γ, transverse to the c_i.

This gives a topology to $\mathcal{WM}$.

The completion of $\mathcal{WM}$ is the space of *measured laminations* $\mathcal{ML}$.



Measured laminations can be pretty complicated.

- $\mathcal{ML} \simeq \mathbb{R}^{6g-6}$.
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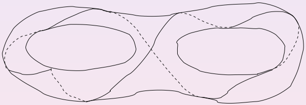

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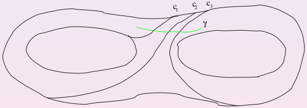

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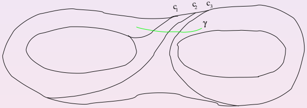

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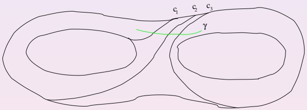

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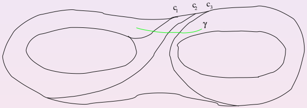

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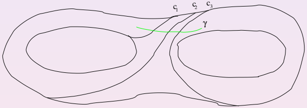

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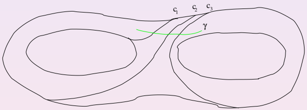

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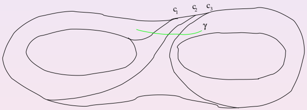

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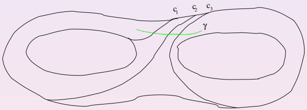

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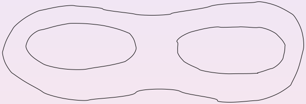
Thurston's Earthquake Thm

Start with a hyperbolic surface.

If $w \in \mathcal{ML}$ is a weighted curve and $h \in \mathcal{T}$, $E_l(w)(h)$ is obtained by realizing w as a geodesic in h, cutting S open along w, turning the left-hand side by the weight, and gluing back.

Defines a homeomorphism

$$E_l(w) : \mathcal{T} \to \mathcal{T} .$$



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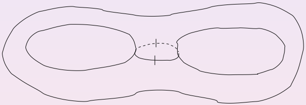

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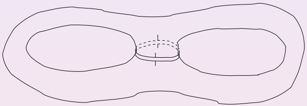

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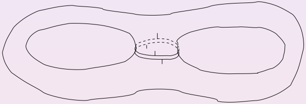

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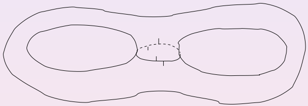

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Start with a hyperbolic surface.
If $w \in \mathcal{ML}$ is a weighted curve and $h \in \mathcal{T}$, $E_l(w)(h)$ is obtained by realizing w as a geodesic in h, cutting S open along w, turning the left-hand side by the weight, and gluing back.
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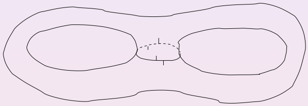

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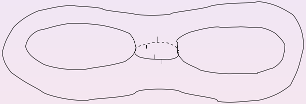

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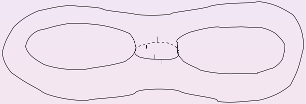

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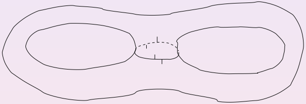

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Extension of the Earthquake Theorem :

- to hyperbolic surfaces with cone sings of angle $< \pi$. (w/ Francesco Bonsante.)
- to hyperbolic surfaces with geodesic boundary : 2^N earthquakes sending h to h'. (w/ Bonsante, Kirill Krasnov).

The proof of the 1st statement is equivalent to an extension of the Mess parameterization for GH AdS manifolds with "particles" : cone singularities along time-like lines, $\theta < \pi$. The analoguous quasifuchsian statement holds : Bers-type theorem for quasifuchsian manifolds with cone singularities of $\theta < \pi$ along infinite lines (Lecuire, Moroianu, S.). The 2nd statement is based on *multi-black holes* : like globally hyperbolic manifolds, based on a complete, non-compact surface. AdS analogs of Schottky mflds.

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A cyclic extension of the earthquake flow

For $\lambda \in \mathcal{ML}$ fixed, $E_l(\lambda)$ defines an action of $\mathbb{R}$ on $\mathcal{T}$, by $(t, h) \mapsto E_l(t\lambda)(h)$. Analog of horocyclic flow.

We define (w/ Bonsante & Gabriele Mondello) an "extension" : equivalently

- for $c \in \mathcal{T}$, $C_c : S^1 \times \mathcal{T} \to \mathcal{T}$,
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- GH AdS 3-mflds,
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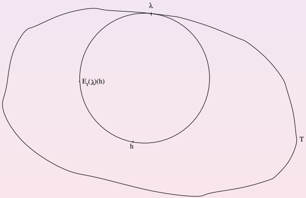

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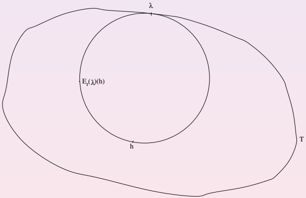

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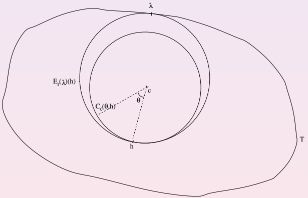

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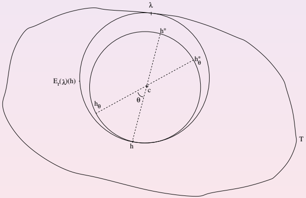

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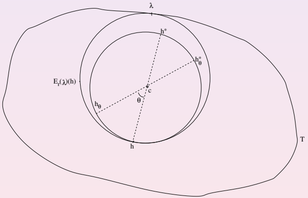

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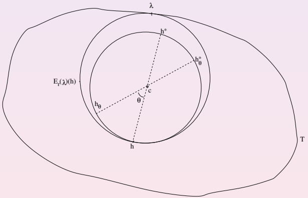

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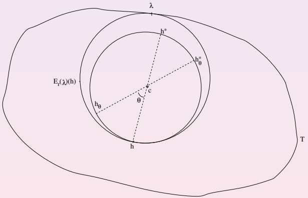

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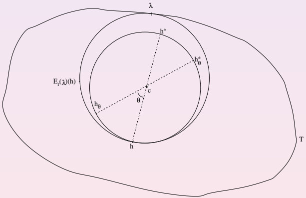

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Properties of the cyclic flow

Some properties :

- Limits to the earthquake flow : if $t_n h_n^* \to \lambda$ then $D_{t_n}(h, h_n^*) \to E_l(\lambda/2)(h)$.
- Extension of the earthquake thm :

$$\forall \theta \in S^1 \setminus \{0\}, \forall h, h' \in \mathcal{T}, \exists! c \in \mathcal{T}, C_c(\theta, h) = h' \ .$$

- Has a complex extension, which limits to McMullen's complex earthquakes.
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The extension of the Earthquake Thm follows from a recent result of Barbot, Béguin and Zeghib on constant Gauss curvature foliations of AdS manifolds.

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A homeo of S^1 is *quasi-symmetric* if it is the boundary of a quasi-conformal diffeo of the disk.

Def. $\mathcal{T}_U$ = space of quasi-symmetric orientation-preserving homeos of S^1, up to $PSL(2, \mathbb{R})$.

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Question : canonical quasi-conformal extension(s) to the disk of a quasi-symmetric homeo ?

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ϕ is min Lagrangian iff $\phi = v \circ u^{-1}$, where $u, v : D \to H^2$ are harmonic maps with opposite Hopf differentials. "Squares" of harmonic map.
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AdS_3 as a Lorentz analog of H^3

$$AdS_3 = \{x \in \mathbb{R}^{2,2} \mid \langle x, x \rangle = -1\} .$$

Constant curvature -1, $\pi_1(AdS_3) = \mathbb{Z}$.

- Conformal model, in a cylinder.
- Projective model, in a quadric.
- Space-like, time-like, light-like directions. Time-like geodesics are closed of length 2π.
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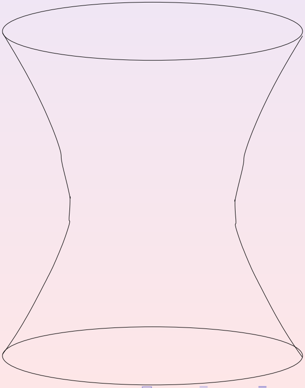

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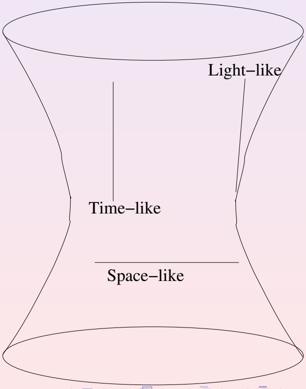

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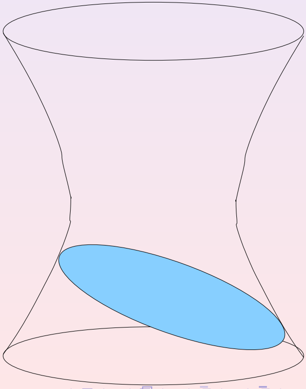

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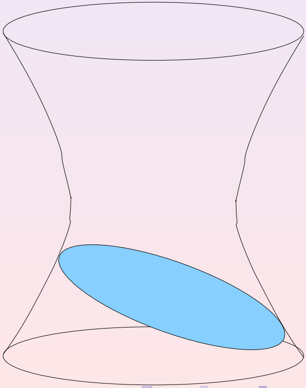

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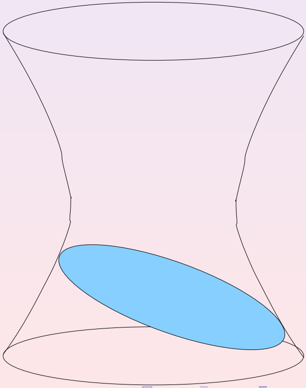

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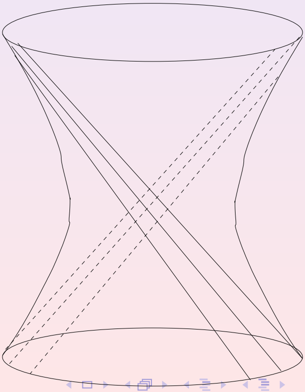

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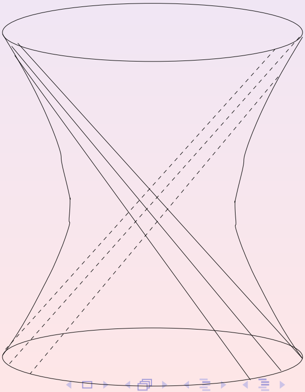

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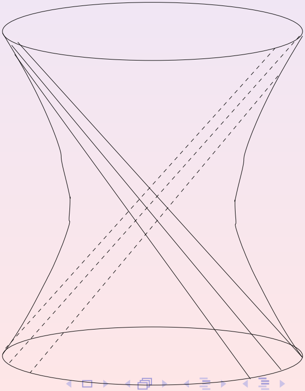

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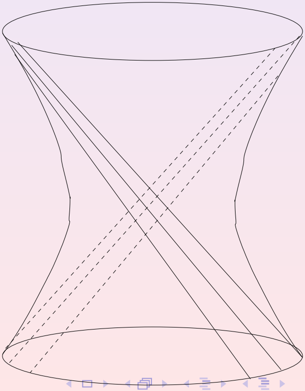

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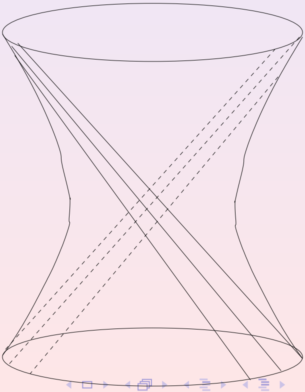

Globally hyperbolic AdS manifolds

Def. an AdS mfld M is *maximal globally hyperbolic* if

- it contains a closed, space-like surface S,
- any inextendible time-like curve intersects S exactly once,
- it is maximal (for inclusion) under those properties.

Then $M \simeq S \times \mathbb{R}$, and $M = \Omega/\rho(\pi_1 S)$, where $\Omega \subset AdS_3$.
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GHMC AdS vs quasifuchsian

M has a "limit set" Λ_Γ, which is a Jordan curve. $\Lambda_\Gamma = \partial\Omega \cap \partial_\infty AdS_3$.

M has a "convex core", $C(M) = CH(\Lambda_\Gamma)/\Gamma$.

It has two boundary components, both with hyperbolic induced metrics $m_\pm$, bent along measured laminations $l_\pm$ that fill (Mess).

Question (Mess). can any $m_\pm$ be uniquely realized ?

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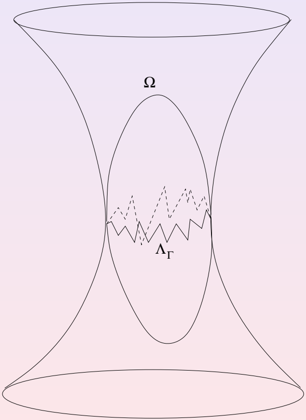

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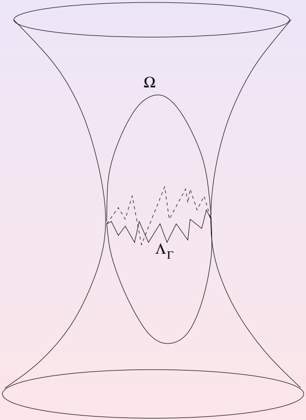

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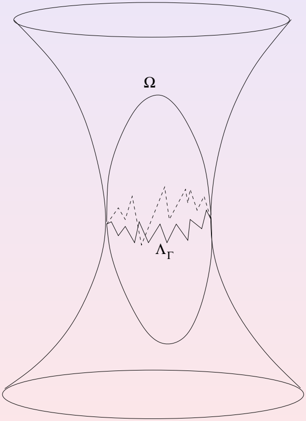

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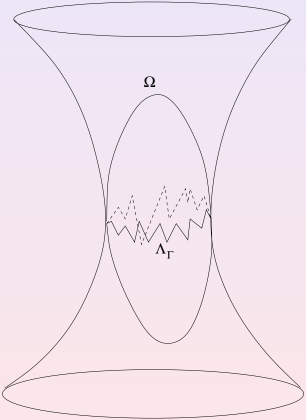

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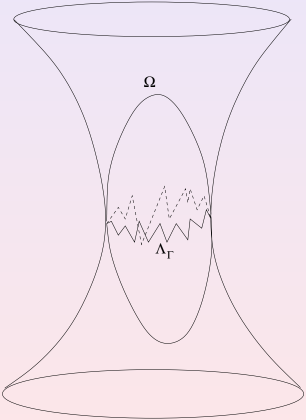

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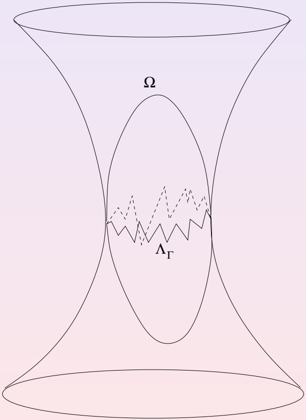

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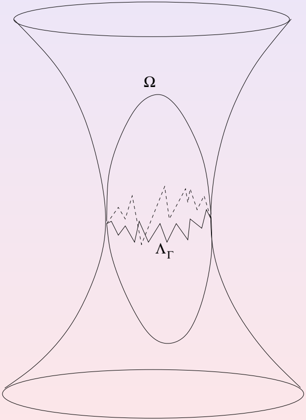

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Given a GHMC AdS mfld M, $\rho : \Gamma \to SO(2,2) \simeq PSL(2,\mathbb{R}) \times PSL(2,\mathbb{R})$.
So, $(\rho_L, \rho_R) : \Gamma \to PSL(2,\mathbb{R})$.
Thm (Mess).

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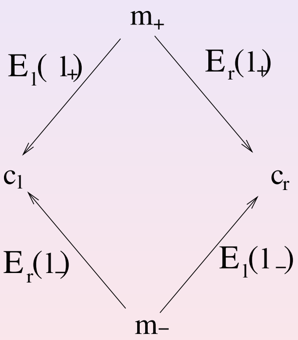

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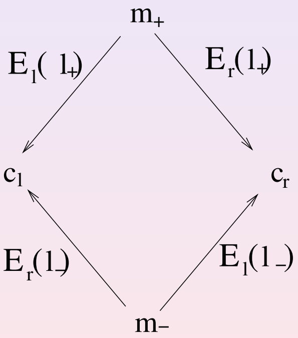

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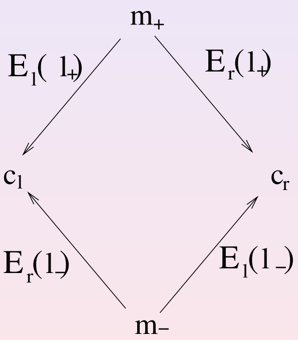

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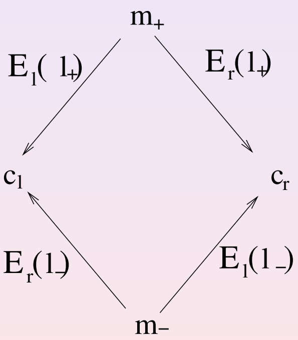

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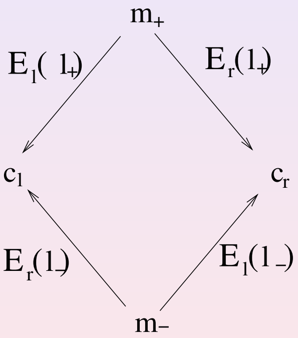

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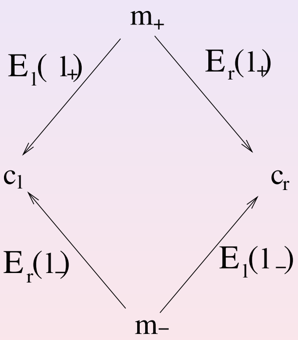

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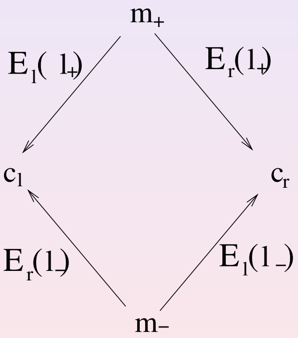

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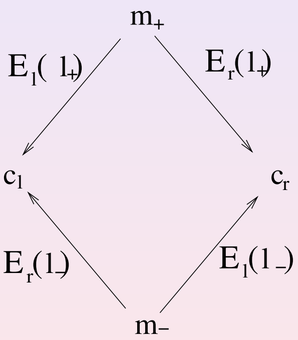

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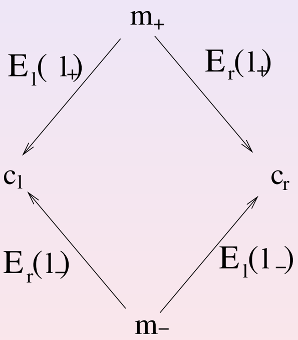

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Maximal surfaces in AdS

Let $\Sigma \subset AdS_3$ be a space-like graph. We call :

- I the induced metric, J its complex structure,
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- E the identity.

Def. $h_L, h_R = I((E \pm JB)\cdot, (E \pm JB)\cdot)$.
Prop (Krasnov, S.). if Σ has principal curvatures $|k_i| < 1$ then h_L, h_R are hyperbolic metrics. If h_L, h_R are complete, we obtain $\phi : H^2 \to H^2$. Related to the left/right representations for GH mflds.
Prop. Σ is *maximal* iff ϕ is min Lagrangian. It is quasi-conformal iff $|k_i| < 1$ uniformly.
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The end

Thanks for your attention !