Convolution inequalities for Boltzmann collision operators and applications

We study integrability properties of a general version of the Boltzmann collision operator for hard and soft potentials in n-dimensions. A reformulation of the collisional integrals allows us to write the weak form of the collision operator as a weighted convolution, where the weight is given by an operator invariant under rotations. Using a symmetrization technique in Lp we prove a Young's inequality for hard potentials, which is sharp for Maxwell molecules in the L^2 case.

Further, we find a new Hardy-Littlewood-Sobolev type of inequality for Boltzmann collision integrals with soft potentials. The same method extends to radially symmetric, non-increasing potentials in some L^s-weak or L^s. The method is an analog a Brascamp, Lieb and Luttinger approach for multilinear weighted convolution inequalities. In all cases, the inequality constants are explicitly given by formulas depending on integrability conditions of the angular cross section (in the spirit of Grad cut-off). As an additional application of the technique we also obtain estimates with exponential weights for hard potentials in both conservative and dissipative interactions.

As an immediate application we obtain that distributional solution of the space inhomogeneous Boltzmann equation for singular (soft) potentials, for initial data near local Maxwellians states and L^a-integrable differential angular cross-section, are classical in the sense that propagate L^p-regularity in physical and velocity space and have L^p-stability for a range of p depending on the space dimension, the potential and angular integrability exponents s and a respectively.

This is work in collaboration with Ricardo Alonso and Emanuel Carneiro.