Pathologies of the Euler equation and its relations with the Navier-Stokes and Boltzmann equation

Claude BARDOS*

Abstract

In spite of the fact that it is an oversimplified model (using only the Euler equation one reaches the conclusion that the birds cannot fly!) it is a corner stone in the mathematical analysis of fluid dynamic... It is, under convenient scaling both the limit of the Navier-Stokes and the Boltzmann equation and the pathologies of its solutions may shed some light on the singular behaviour (for instance in relation with turbulence) of the solutions of these "more realistic" equations. In consequence I will organi

1 Lecture 1. Hierarchy of equations and instability of the Euler equation

I will shortly describe a hierarchy of equation which starts from the Newtonian mechanic and ends with the models of turbulence. The goal is to show that the Euler equation is in the center of this hierarchy. Then I will show on simple and example the instabilities of the solutions of the $3d$ Euler equation.

2 Lecture 2 and 3 The wild solutions of De Lellis and L. Szkelyhidi

The existence of very singular weak solutions of the Euler equation has already been observed by Scheffer and Shnirelman which in particular are of space-time compact support. However recently De Lellis and Szkelyhidi gave a construction of a residual set of such solutions inspired by the Nash Kuiper theorem in Riemannian geometry and using the differential inclusions of Tartar.

3 Lecture 4.The derivation of the Euler Equation from Boltzmann equation in the Incompressible limit following Laure Saint Raymond

This very elegant derivation is coherent with the fact that coarser limits can be obtained whenever the properties of the limit model are well established. More precisely the result of Laure Saint Raymond is valid as long as the solution of the corresponding Euler equation remains smooth.

*Emeritus Professor Laboratory Jacques Louis Lions University Pierre et Marie Curie Paris. (claude.bardos@gmail.com)

References

[1] C. Bardos and E. Titi "Euler equation for incompressible ideal fluids." Uspeki Math. Nauk 62:3, 5-46.

[2] C. Bardos and E. Titi, "Loss of smoothness and energy conserving rough weak solutions for the 3d Euler equations" arXiv:0906.2029.

[3] De Lellis and L. Szkelyhidi " On admissibility criteria for weak solutions of the Euler equations" arXiv:0712.3288.

[4] B. Mohammadi and O. Pironneau "Analysis of the K-Epsilon Turbulence Model", Wiley Masson, 1994.

[5] L. Saint Raymond, "Convergence of Solutions to the Boltzmann Equation in the Incompressible Euler Limit", *Arch. Rational Mech. Analysis*, **166** (2003), 47–80.

[6] "PARADOXE DE SCHEFFER?SHNIRELMAN REVU SOUS L?ANGLE DE L?INTE?GRATION CONVEXE" d?apr'es C. De Lellis et L. Sz?ekelyhidi S?eminaire BOURBAKI Novembre 2008 61'eme ann?ee, 2008-2009, no 1001.