Shallow Water Circulation Model

In this talk I'll discuss 2-Dimensional flows for the "shallow-water" equations. The model describes the motion of a finite volume of fluid taking place in container whose bottom is described by a paraboloidal like surface of the form:

$$z = (\alpha x^2 + \beta y^2)/2, \alpha > 0, \beta > 0$$

or more generally

$$z = a(x, y)$$

where a tends to infinity as $(x^2 + y^2)$ tends to infinity. The model includes gravity, coriolis, and viscous forces.

I'll discuss a set of a-priori estimates and other key facts about the solution of the viscous shallow-water system. I'll also present an interesting exact Rotating-Pulsating solution to this system.

Such Rotating-Pulsating solutions are well known for inviscid shallow-water equations.

I'll also present a Lagrangian reformulation of the shallow-water system. This formulation holds in a fixed spatial domain which corresponds to the region in space which is "wet" at time t = 0. It represents a somewhat more involved system of PDE's than the original Eulerian formulation but with this formulation we avoid having to explicitly track the unknown free-boundary where the height of the water column vanishes.

The Lagrangian model induces a natural computational model. I'll discuss briefly the parallel implementation of the computational model and present performance results and timing comparisons for the parallel implementation of the computational model; our speedups are impressive.

I'll conclude with two simulations which demonstrate that Lagrangian computational model produces steady-state or long-time solutions like the exact Rotating-Pulsating solutions discussed earlier. These results validate the effectiveness of the Lagrangian reformulation of the problem.