

Global structure stability of Riemann solutions for linearly degenerate hyperbolic conservation laws under small BV perturbations of the initial data

Zhi-Qiang Shao

Department of Mathematics, Fuzhou University, Fuzhou 350002, China

E-mail: zqshao-fzu@yahoo.com.cn

Abstract: In this paper, we study the global structure stability of the Riemann solution $u = U(\frac{x}{t})$ for general $n \times n$ quasilinear hyperbolic systems of conservation laws under a small BV perturbation of the Riemann initial data. We prove the global existence and uniqueness of piecewise C^1 solution containing only n contact discontinuities to a class of the generalized Riemann problem, which can be regarded as a small BV perturbation of the corresponding Riemann problem, for general $n \times n$ linearly degenerate quasilinear hyperbolic system of conservation laws; moreover, this solution has a global structure similar to the one of the self-similar solution $u = U(\frac{x}{t})$ to the corresponding Riemann problem. Our result indicates that this kind of Riemann solution $u = U(\frac{x}{t})$ mentioned above for general $n \times n$ quasilinear hyperbolic systems of conservation laws possesses a global nonlinear structure stability under a small BV perturbation of the Riemann initial data. Applications include the one-dimensional Born-Infeld system arising in the string theory and high energy physics.

References

- [1] A. Bressan, *Hyperbolic Systems of Conservation Laws: The one-dimensional Cauchy Problem*, Oxford University Press, Oxford, 2000.
- [2] A. Bressan, Contractive metrics for nonlinear hyperbolic systems, *Indiana Univ. Math. J.* 37 (1988) 409-421.
- [3] D. X. Kong, Global structure stability of Riemann solutions of quasilinear hyperbolic systems of conservation laws: shocks and contact discontinuities, *J. Differential Equations* 188 (2003) 242-271.
- [4] P. D. Lax, Hyperbolic systems of conservation laws II, *Comm. Pure Appl. Math.* 10 (1957) 537-556.
- [5] T. T. Li, D. X. Kong, Global classical discontinuous solutions to a class of generalized Riemann problem for general quasilinear hyperbolic systems of conservations laws, *Comm. Partial Differential Equations* 24 (1999) 801-820.
- [6] T. T. Li, W. C. Yu, *Boundary Value Problems for Quasilinear Hyperbolic Systems*, Duke University Mathematics Series V, Duke University, Durham, 1985.

- [7] Z. Q. Shao, Global structure stability of Riemann solutions for general hyperbolic systems of conservation laws in the presence of a boundary, *Nonlinear Anal.* 69 (2008) 2651-2676.