Some Mathematical Aspects of a Dirac Particle Interacting with the Quantum Radiation Field

Asao Arai
Department of Mathematics, Hokkaido University
Sapporo 060-0810
Japan
E-mail: arai@math.sci.hokudai.ac.jp

Abstract

We consider a quantum system of a Dirac particle interacting with the quantum radiation field, whose Hamiltonian is given by $H := H_D(V) + H_{\rm rad} - q\sum_{j=1}^3 \alpha_j A_j(x)$ ($x \in \mathbb{R}^3$), where $H_D(V)$ is the Dirac operator with V a 4×4 Hermitian matrix-valued potential on $\mathbb{R}^3$, $H_{\rm rad}$ is the free Hamiltonian of the quantum radiation field, $q \in \mathbb{R} \setminus \{0\}$ denotes the electric charge of the Dirac particle, $\alpha_j, j = 1, 2, 3$, are the Dirac matrices and $(A_1(x), A_2(x), A_3(x))$ is the quantum radiation field in the Coulomb gauge with momentum cutoffs. After reviewing basic properties of the operator H including essential self-adjointness and spectral properties, we present some results on the Heisenberg operators $x_j(t) := e^{it\bar{H}} x_j e^{-it\bar{H}}$ of the j-th position operator x_j of the Dirac particle ($t \in \mathbb{R}$, $j = 1, 2, 3$; $\bar{H}$ is the closure of H) and their derivatives in t, the velocity operators. An effect of the quantum radiation field to the usual Zitterbewegung of the Dirac particle is described.