IS THERE A PHILOSOPHICAL VIEW ALREADY IN MATHEMATICAL LOGIC

In Mathematical Logic,
there are two ways we
encounter notions of "grounds":

Proof Theory:
metastructure N, T Σ over N,
N via T grounds
a N-model of T

Model Theory:
in model M,
a base set $\bar{c}$ grounds
elements $a, b, \ldots$ in M:
$tp(a/\bar{c})$, $tp(b/\bar{c})$.

Here is the
view which is
(essentially) already inside
Mathematical Logic:

Take what is grounded
in the model theoretic sense,
and move it into the
meta structure,
thus obtaining:

that which is in need of
(model theoretical grounding

becoming

that which gives grounds
(proof theoretic)

to

its own (needed) grounding
(model theoretical

Plan of this talk:

① present: what the above
"the grounded-giving-grounds-for-its-own-grounding"
means (you already essentially know of this — it is (essentially) already in mathematical logic)

② Suggest: ① can make sense as applying to "us" (to "us" as mortals, but, for this talk, to "us" as mathematicians)

③ Start: elaborating (while still staying inside mathematical logic), while also

start: mentioning certain philosophers with the suggestion that what is being elaborated bears a resemblence to what these philosophers say

Concerning 2:
[2: suggesting that this applies
to "us" as mathematicians]:
this talk adopts the attitude that
abstract model theory can apply
to "us"; whatever happens abstractly
to "us"
in all models happens

Concerning 3:
[3 elaborating while suggesting
resemblances to what certain
philosophers say]:

it is often controversial and/or
obscure as to
what certain philosophers say;

this talk adopts the attitude
of presuming that these
philosophers are speaking of things
that resemble the 1 of this talk,
and from that presumption noting
the continuation of resemblances

As initial instances of 3:

Parmenides: "resembling a well-rounded sphere equal in all ways from the center" (model theoretic grounding)

Heidegger: "the possibility of the impossibility of the existing" (proof theoretic grounding)

A personal note:

the issues of this talk
arose from a deeply felt
personal need inside myself –
the need to hear something
coming from deep inside myself.

So, to you as mathematicians:

this talk is not intended
to convince you of anything
(which would involve your
ability to judge the correctness
of things)

this talk is intended
to simply speak to you of something
(which does involve your
ability to hear meaningful things)

Metatheory:
Σ-bounding

$N \models \Sigma$-bnd : N is to be thought of:
(as end extendable to N' and so
N is $< n'$ (n' in N'))
N as part of something (n')
which is viewed as finite.

Σ-bnd says that the proof theory of
N corresponds to model theory:

for T Σ over N,

T consistent (no proof p in N
p proof of a
contradiction from T)

iff

there is a model M of T
for N

(Here, N and M are assumed to
be countable)

M a N-model for T means:
there is N-truth set for M:

$$\{\langle \varphi, \bar{a}\rangle \mid \bar{a} \text{ in } M^i\ (i \in N),\ M \models \varphi(\bar{a}),\ \varphi \text{ in } \mathcal{L}^N\}$$

If we view N as listing $\mathcal{L}$, (the language)
$\mathcal{L} = \bigcup_{i \text{ in } N} \mathcal{L}_i$, then

we can form

$$L_N[M] = \bigcup_i L_i[M]$$

$L_0[M]$ = something interpretable from $M|\mathcal{L}_i$

and $L_{i+1}[M] = \begin{pmatrix} i\text{-complexity definable} \\ \text{subsets of } L_i[M] \end{pmatrix} \cup L_i[M]$

M (as set) is in $L_0[M]$
$M|\mathcal{L}_i$ is in $L_N[M]$.

$L_N[M]$ is analogous to the construction $L_\alpha[M]$

T N-consistent iff
T has a Σ set-model M for N

M Σ-set for N
iff
$L_N[M] \models \Sigma$-bnd

So,
$L_N[M]$ is itself a suitable meta structure

Proof theory over $L_N[M]$ allows the M-rule:
from $\varphi(a)$ (each a in M)
infer $\forall x \varphi(x)$

Or, equivalently,
$L_N[M]$ allows for formulas built by $\bigvee_a$, $\bigwedge_a$ (over a in M)

Remark:

If M is $\underset{\sim}{\Sigma}$-set for N
then $L_N[M]$ will have end extension
$L_{N'}[M']$ $(M' \restriction \mathcal{L}^N = M)$

Conversely,
if for $N' \in$ some N', $N' > N$,
$L_{N'}[M']$ exists, then
$M = M' \restriction \mathcal{L}^N$ must be $\underset{\sim}{\Sigma}$-set for N.

So
the assumption
M is $\underset{\sim}{\Sigma}$-set for N
just corresponds to the areas
of N being part of something finite
where M itself is part of
that area. by simply
being $\underset{\sim}{\Sigma}$-set

Finitism (in Mathematical Logic)
allows for $N \models \Sigma$-based
as meta structures.

Platonism (in Mathematical Logic)
allows for actual structures, MET

The $L_N[M]$ (M Σ-set)
combine these.

From this perspective,
the issue
Finitism verses Platonism

can be perhaps phrased
as the issue:

Can some M be "finitary" enough
so $L_N[M]$ can serve as a
metastructure?

That this question cam accomodate a notion of "finitary" beyond the notion of N-finite is pretty cleer:

N extends to N'

If M' is N'-finite (ie M' is in N')

then $M = M' \cap L^N$ would seem to be "finitary".

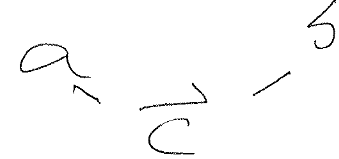
(ie $L_N[M] \models$ "M is possibly finite")

But, is there a way to think of "finitary" that doesn't tie ~~it~~ so directly to the notion of finite?

Let's now look at Model Theory:

What do we know about what can be done in a model M even when we know nothing about M?

Always have the patterns



($\vec{c}$ is the ground for how a and b are relating)

$tp(a, b / \vec{c})$ provides the means by which

$tp(a / b\vec{c})$ and $tp(b / a\vec{c})$

belong together.

Let's consider languages $\mathcal{L}$ that allow us (in a first order way) to talk about something like this!

Consider $\mathcal{L}^N$ that include predicates I_i ($2i$-ary) intending to capture the full back & forth:

$$I_i(\bar a, \bar b) \equiv \begin{matrix}\forall a \exists b \\ \& \\ \forall b \exists a\end{matrix} \; I_{i+1}(a, \bar a; b, \bar b)$$

$$I_i(\bar a, \bar b) \Rightarrow I_j(\bar a \restriction s, \bar b \restriction s)$$

$$I_i(\bar a, \bar b) \Rightarrow (\varphi(\bar a) \Leftrightarrow \varphi(\bar b))$$
(each φ in $\mathcal{L}^N$)

For $M \models$ this we get

$$\mathrm{Aut}(M) = \{\sigma \mid \sigma : M \to M \;\; I_i(\vec a, \sigma \vec a) \}$$
(all $\vec a$) (all $i \in N$)

(~~the~~ the N-truth set for M provides us with M^i ($i \in N$)

for $\bar a$ from M^i, we require that $\sigma(\bar a)$ is in M^i)

Remark:

$I_i(\bar{a}, \bar{b})$ never can catch-up to its intended meaning of

$$tp_{L^N}(\bar{a}/\emptyset) = tp_{L^N}(\bar{b}/\emptyset)$$

At best it refers to some N' extending N & n' in N s.t. $\bar{a}$ & $\bar{b}$ are equivalent up to n'
(ie $tp_{L^{N'}_{n'}}(\bar{a}) = tp_{L^{N'}_{n'}}(\bar{b})$)

EG: one way to get such predicates I_i for an M:

Do the back+forth predicates for N':

$$(BF)_{i,k}(\bar{a}, \bar{b}) \equiv \begin{array}{c}\forall a \exists b \\ \& \\ \forall b \exists a\end{array} BF_{i+1,k-1}(a, \bar{a}; b, \bar{b})$$

$$BF_{i,0}(\bar{a}, \bar{b}) \equiv \bigwedge_{\substack{\varphi \in L_i \\ \varphi \text{ atomic}}} (\varphi(\bar{a}) \leftrightarrow \varphi(\bar{b})).$$

And let $I_i(\bar{a}, \bar{b}) \equiv BF_{i, n'-i}(\bar{a}, \bar{b})$.

So, $I_i(\bar{a}, \bar{b})$ is intended to be refering to the possibility of an extension N' of N and a fragment $L_{N'}^{N'} = F$ s.t. $tp_F(\bar{a}/\emptyset) = tp_F(\bar{b}/\emptyset)$.

So, $tp_F(b/a\bar{c})$ corresponds to

$$Q_{b,a,\bar{c}} = \{ b' \in M \mid M \models I_{i+2}(b, a, \bar{c}; b', a, \bar{c}) \}$$

where $Q_{b,a,\bar{c}}$ is given the inherited structure:

$$I_j^Q(\bar{b}, \bar{b}') \equiv I_{i+1+j}(\bar{b}, a, \bar{c}; \bar{b}', a, \bar{c}).$$

Given M, given $\langle\langle a_i, b_i, \vec{c}_i\rangle, i<r\rangle$ from M ($r \in N$)

let $Q_i = Q^M_{b_i, a_i, \vec{c}_i}$

Form $Q_0 \cup \ldots \cup Q_{r-1} = B$.

B is interpretable in M.

So, can form $L_N[B] \models \Sigma$-bnd

Conversely, given such a B ($B = Q_0 \cup \ldots \cup Q_{r-1}$, $r \in N$)

B Σ^N-saturated,

Over $L_N[B]$, can form Σ-theories of the form

$T \cup$ "$f_j : Q_{i_j} \cong Q^M_{b_j, a_j, \vec{c}_j}$" ($j < t$; t in N)

(where T is a Σ theory of $\mathcal{L}^N(\langle a_j, b_j, \vec{c}_j\rangle, j<t)$ (new constants)

and the f_j are new function symbols)

Call theories of this form

(the form: $T \cup$ " $f_j : Q_{i_j} \xrightarrow{\sim} Q^{M}_{b_j, a_j, \bar{c}_j}$ ")

<u>positioning</u> <u>theories</u>.

The net effect:

Have $L_N[B] \models \Sigma$-bnd

and so $L_N[B]$ can serve as a proof theoretic ground;

and yet, positioning theories express that B is in need of model theoretic grounding, and, a model M of such theories fulfills that need.

The preceeding is (in a sense) a "view" which (in a sense) is "already in mathematical logic". (end of part 1)

Do we have a "philosophical view" here? (start of part 2)

Perhaps if we add the intention to a $B = Q_0 \cup \dots \cup Q_{r-1}$ that each Q_i is, intentionally, about "us" mathematicians:

is about $Q^M_{a,b,\vec{c}}$ where a & b are ~~us~~ instances of "us".

From the point of view
of model theory,
we don't need to know anything
about "us" to know that
"us" in a model M will be
just as things always are in
a model:

If "we" are interacting with
objects $\bar{c}$, and "we" let
$\bar{c}$ ground "us", then we get
such positioned Q's,

intuitively,
Q = how b is to a
as grounded by $\bar{c}$.

a Q = how we are to ourselves
as grounded by some thing

Such B ($B = Q_0 \, i \ldots i \, Q_{r-1}$)
can also come from models
of "real" situations involving "us":

$M_R \models T^R$

While also coming from models
of "imaginary" mathematical
situations in which we let "us"
also appear:

$M_\partial \models T^\partial$

If $L_N[B]$ extends to $L_{N'}[B']$
($B' \upharpoonright \{I_i \mid i \text{ in } N\} = B$)
where T^∂ is inconsistent in N'
but T^R is still consistent in N'
then B (or rather B extended to B')
is still there, even though it can
no longer be positioned in a model
as specified by T^∂

When is a $\mathcal{B}$ finitary?

One suggestion:

$\mathcal{B} = Q_0 \cup \cdots \cup Q_n$ is finitary over $\mathcal{N}$

iff for each Q_i,
there is a real T^R with
real model $\mathcal{M} \models T^R$ positioning
Q_i as $Q^{\mathcal{M}}_{a,b,\bar{c}}$, where
$a, b, \bar{c}$ is from ordinary life.

(Notice, $Q^{\mathcal{M}}_{a,b,\langle\rangle}$ is not finitary)

(end of part 2)

start of part 3:

The proof-theoretic grounding provided by

$\mathcal{L}_N[B] \quad (B = Q_0 \, i \ldots i \, Q_{r-1})$

involves "us" mainly as a (the) 6 in each Q_i.

For p in $\mathcal{L}_N[B]$ a proof, the one who can comprehend the proof p is not "us" in the sense of (the) 6 in Q_i's; "we" are in the Q_i, and so we are integral to providing the Q_i's;

but "we" cannot survey the $Q_i \restriction \mathcal{L}_n$ $(n \in N)$, and that is what is required to comprehend a proof p.

(resemblance to Heidegger)

Full freedom in imaginary T^d:

means, by def,

for the given $B = Q_0 \cup \dots \cup Q_{r-1}$

all the positionings of all the Q_i

fit together as coming from

(the Aut(M)-orbit of) one a, b:

for all positionings $a_j, b_j, \bar{c}_j$,

$I_4(a, b; a_j, b_j)$ (ie $\sigma(a,b) = (a_j, b_j)$ for some $\sigma \in Aut(M)$):

there is only one relevant way a mathematician a gives to another mathematician b,

and only one way b receives from a.

(resemblance to Parmenides)