

REVERSE MATHEMATICS AND IRREGULAR PRINCIPLES

DAMIR D. DZHAFAROV

Reverse mathematics is an area of mathematical logic and foundations devoted to classifying mathematical theorems according to their logical strength, as measured using subsystems of second-order arithmetic. It is a common intuition among mathematicians that some theorems in disparate areas of mathematics are deeply interrelated, and that the same patterns of problem-solving crop up in solutions to otherwise seemingly unrelated problems. Reverse mathematics can be viewed as a formalization and confirmation of this intuition. Indeed, a striking fact repeatedly demonstrated by research in reverse mathematics is that a vast number of mathematical theorems can be classified into just five main types, either being provable in the base subsystem RCA_0 , or else being equivalent over RCA_0 to one of WKL_0 , ACA_0 , ATR_0 , and $\Pi_1^1\text{-CA}_0$. I will discuss some recent research on mathematical principles that fall outside of this framework, and whose logical strength is therefore “irregular” and more difficult to understand. These include many important mathematical results from combinatorics, model theory, and set theory, including versions of Ramsey’s theorem, the atomic model theorem, and the finite intersection principle. I will begin with a brief introduction to reverse mathematics, and conclude with some open problems concerning irregular principles.