

Iteration trees

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We shall cover the basic theory of iteration trees, and some of its applications. We shall begin at a basic level, with ultrapowers of models of ZFC, and try as we proceed to keep the course accessible to anyone who has taken a graduate-level course in set theory. (In particular, no background in inner model theory will be assumed.)

Here is a rough outline of the course:

1. ultrafilters, ultrapowers, extenders, iterated ultrapowers, linear iterability. (See for example [1].)
2. Some applications: $P(\omega) \cap L$ is countable, generic absoluteness for Σ_3^1 formulae. (See [1].)
3. Iteration trees. Existence and uniqueness for cofinal wellfounded branches in iteration trees. Iteration strategies. Constructing iteration trees via the one-step lemma. ([3], [5], [4], [2].)
4. Homogeneously Suslin sets, Projective Determinacy.([2], [4].)
5. A comparison lemma for coarse inner models.([3])
6. The Extender Algebra, and iterations to make reals generic. Generic absoluteness from iterable models with Woodin cardinals.(See for example [6], section 7.)
7. Counterexamples to the Unique Branches Hypothesis and to the Cofinal Branches Hypothesis.

There is probably more here than we can cover in the allotted time. We shall adjust the speed to the audience, and omit or merely sketch some of the material in (4)–(7) as required.

References

- [1] Thomas Jech, *Set Theory*. Springer Monographs in Mathematics. Springer, Berlin, 2002.
- [2] D.A. Martin and John Steel, *A proof of projective determinacy*. Journal of the AMS, **2**, 1989, 71-125.
- [3] D.A. Martin and John Steel, *Iteration trees*. J. of Amer. Math. Society, **7**, 1994, 1-73.
- [4] Itay Neeman, *Determinacy in $L(\mathbb{R})$* . in Handbook of Set Theory, volume 3, M. Foreman and A. Kanamori eds., Springer, 2010.
- [5] John Steel, *The wellfoundedness of the Mitchell order*. J. of Symbolic Logic, **58**, 1993, 931-940.
- [6] John Steel, *An outline of inner model theory*, in Handbook of Set Theory, volume 3, M. Foreman and A. Kanamori eds., Springer, 2010.