Solving word problems in group extensions over infinite words

In my lecture I report on a joint work with Alexei G. Myasnikov.

Non-Archimedean words have been introduced as a new type of infinite words which can be investigated through classical methods in combinatorics on words due to a length function. The length function, however, takes values in the additive group of polynomials $Z[t]$ (and not, as traditionally, in N), which yields various new properties. Non-Archimedean words allow to solve a number of algorithmic problems in geometric and algorithmic group theory. There is a connection to the first-order theory in free groups (Tarski Problems), too.

We provide a general method to use infinite words over a discretely ordered abelian group as a tool to investigate certain group extensions for an arbitrary group G. The central object is a group $Ext(A, G)$ which is defined in terms of a non-terminating, but confluent rewriting system. The group G as well as some natural HNN-extensions of G embed into $Ext(A, G)$ (and still "behave like" G), which makes it interesting to study its algorithmic properties.

The main result characterizes when the Word Problem is decidable in all finitely generated subgroups of $Ext(A, G)$.

Our methods combine combinatorics on words, string rewriting, and group theory.