Cyclotomic constructions of strongly regular Cayley graphs and difference sets

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The idea of constructing difference sets and strongly regular Cayley graphs from cyclotomic classes of finite fields goes back to Paley. In the mid-20th century, this idea was pursued vigorously by many researchers, such as Baumert, Chowla, Hall, Lehmer, van Lint, Schrijver, Storer, Whiteman, Yamamoto, etc. However, this method for constructing difference sets has had only very limited success. Let q be a prime power and $N|(q-1)$, $N > 1$. It is known that a single cyclotomic class of order N can form a difference set in $(\mathbb{F}_q, +)$ if $N = 2, 4$, or 8 and q satisfies certain conditions. It was conjectured that the converse is also true. Namely, if the nonzero Nth powers of $\mathbb{F}_q$ form a difference set in $(\mathbb{F}_q, +)$, then N is necessarily $2, 4$, or 8. This conjecture has been verified up to $N = 20$. There is a conjecture of similar nature for cyclotomic strongly regular graphs.

We will report new constructions of both strongly regular Cayley graphs and skew Hadamard difference sets by using unions of cyclotomic classes of large orders. The main tools we used are index 2 Gauss sums, instead of cyclotomic numbers. The talk is based on joint work with Tao Feng.