A New Approach to Permutation Polynomials over Finite Fields

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Abstract

Let p be a prime and q a power of p. We study the permutation properties of the polynomial $g_{n,q} \in \mathbb{F}_p[\mathbf{x}]$ defined by the functional equation $\sum_{a \in \mathbb{F}_q}(\mathbf{x} + a)^n = g_{n,q}(\mathbf{x}^q - \mathbf{x})$. The polynomial $g_{n,q}$ is a q-ary version of the reversed Dickson polynomial in characteristic 2. We are interested in the parameters $(n, e; q)$ for which $g_{n,q}$ is a permutation polynomial (PP) of $\mathbb{F}_{q^e}$. We find several families of such parameters and obtain various necessary conditions on such parameters. The results obtained so far, both theoretical and numerical, indicate that the class $g_{n,q}$ contains an abundance of PPs over finite fields, many of which are yet to be explained and understood. The purpose of this talk is to bring up to date what is known about the permutation properties of $g_{n,q}$.