A diffusion approximation for Hubbell's local community model.

G. Espinoza[1], A. Ganz[2] & P.A. Marquet[1]

Department of Ecology, Pontificia Universidad Católica, Chile, CASEB-IEB;

Department of Mathematic, Pontificia Universidad Católica, Chile, ANESTOC Laboratory.

Abstract

In the last years different formalisms have been proposed in order to obtain an analytic explicit solution for the Hubbel neutral model of biodiversity [4]. In this context, several authors [10, 7, 9] have proposed different probability distributions to explain the relative species abundance (RSA) distribution in local community context. However, none of them have derived a simpler formula comparable with the well known Preston lognormal distribution [8]. Here we propose a continuous diffusion approximation process for the Hubbell's model in which the transitions birth-death rates follow a herding asymmetric model. This allow us to obtain an stochastic differential equation with both drift and diffusion terms following the Kimura scheme for neutral genetics models [5], and we are able to deduce a general expression for the RSA distribution for all time t. Finally, at the equilibrium, we find an asymmetric beta distribution depending only in one parameter as a function of the number of species S and the migration m. Additionally, we show that this simpler beta distribution fits equally well as the Volkov's distribution applied on the classical tropical forests database [2].

Keywords: Neutral Theory, Diffusion approximation, Ecological assembled communities, Population dynamics, Beta distribution.

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