Wrinkling Phenomena in Thin Film/Substrate Systems

Wei Jiang[1]

[1]Department of Mathematics, National University of Singapore

Abstract

In this talk, we talk about the wrinkling phenomena in thin film/substrate systems at the micro-nano scale by physical modelling, theoretical analysis and numerical simulations. Our work mainly includes two parts.

In the first part of the talk, we talk about the wrinkling phenomena of a compressed anisotropic elastic thin film bonded to a viscous layer. In the part, to further explore the underlying mechanisms of the role played by the anisotropy in the wrinkling evolution process, we generalize Huang-Suo model to a $1+2$ dimensional mathematical model for anisotropic elastic thin films with cubic crystalline symmetry, by combining the FvK plate theory and the Reynolds lubrication theory. Linear stability analysis (LSA) shows that, during the initial growth stage, for materials with cubic crystalline symmetry, the sign of the degree of elastic anisotropy $\zeta = \frac{C_{12}+2C_{44}}{C_{11}} - 1$ plays an important role in the anisotropy of the buckling instability of the thin film system. More precisely, the growth rate of the fastest growing wave number, taking as a function of directions, reaches a peak in the $< 100 >$ directions for $\zeta > 0$, and in the $< 110 >$ directions for $\zeta < 0$.

To explore the long time evolution behavior of the wrinkling pattern, a highly efficient semi-implicit Fourier spectral algorithm is developed. The numerical experiments show that the wrinkling evolution process can be separated into three main stages: the first is the filtering stage in which high frequency components are filtered out; the second is the initial growth stage in which the average wavelength drops slightly and the wrinkle amplitude grows exponentially; the third is the coarsening stage in which the amplitude as well as the average wavelength of the wrinkles increase constantly and exhibit a power law scaling. Moreover, numerical experiments show that the wrinkle pattern evolution at the initial growth stage can be well predicted by LSA. Numerical experiments for the coarsening stage also reproduce the main features of the pattern formation and evolution observed in physical experiments.

In the second part of the talk, we talk about the pattern formation of a compressed elastic film on a soft elastic substrate by dynamical evolution method. By

generalizing the physical model, LSA and numerical algorithm presented by the first part to study the wrinkling evolution of an anisotropic elastic thin film on a incompressible viscoelastic layer, we succeed in acquiring some wrinkle patterns observed in physical experiments. Furthermore, by numerical experiments we find that under the equi-biaxial condition of initial residual stresses, when its amplitude is slightly larger than the critical stress, the checkerboard pattern can occur and its orientation depends on the sign of the anisotropic degree of the elastic thin film with cubic crystalline symmetry. Moreover, its occurrence is very sensitive to the amplitude of the initial residual stress, and our above numerical experiments are consistent with Audoly's theoretical analysis. Finally, we study the influence of initial residual stress loading history on the wrinkle pattern formations. By the numerical experiments, we observed that the transition from strip patterns to 2D herringbone patterns, and succeed in reproducing the main features observed by Lin and Yang's physical experiment.