A Superfluid Universe

Lecture 2
Quantum field theory & superfluidity

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Lecture 2.

- Quantum fields
- The dynamical vacuum
- Vacuum scalar field
- Superfluidity
  - Ginsburg-Landau theory
  - BEC
- Quantized vortex
  - Vortex dynamics
  - Quantum turbulence

Quantum field: An operator attached to each point of space-time:

$\phi(\mathbf{r}, t)$	Annihilates 1 particle at ***r,t***	Non-relativistic
	Annihilates 1 particle, or creates 1 antiparticle, at ***r,t***	Relativistic

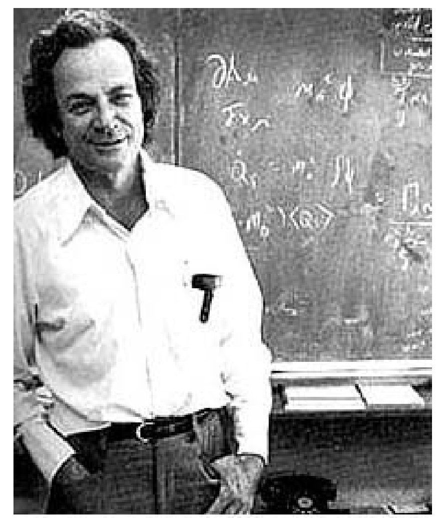
$N = \int d\mathbf{r}\phi(\mathbf{r}, t)^{\dagger}\, \phi(\mathbf{r}, t)$ = (No. of particles) – (No. of antiparticles)

Time translation is represented by phase factor exp(-*iEt*).

$E = \frac{p^2}{2m}$ Non-rel. : only positive frequencies

$E = \pm\sqrt{p^2 + m^2}$ Relativistic: Positive and negative f

Antiparticle = Particle going backwards in time.
--- Feynman



Richard P. Feynman
1918-1998

Quantization: initial condition at *t=0*:

$$\phi(\mathbf{r}) = \begin{cases} L^{-3/2} \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} a_{\mathbf{k}} & \text{(Non-rel)} \\ L^{-3/2} \sum_{\mathbf{k}} [e^{i\mathbf{k}\cdot\mathbf{r}} a_{\mathbf{k}} + e^{-i\mathbf{k}\cdot\mathbf{r}} b_{\mathbf{k}}^{\dagger}] & \text{(Rel)} \end{cases}$$

$$\mathbf{k} = \frac{2\pi}{L}\mathbf{n} \quad \left(\text{Components of } \mathbf{n} = 0, \pm 1, \ldots\right)$$

Annihilation and creation operators:

$$[a_{\mathbf{k}}, a_{\mathbf{p}}^{\dagger}]_{\pm} = \delta_{\mathbf{kp}}$$

(+ fermion - boson)

$$[b_{\mathbf{k}}, b_{\mathbf{p}}^{\dagger}]_{\pm} = \delta_{\mathbf{kp}}$$

$$[a_{\mathbf{k}}, b_{\mathbf{p}}^{\dagger}]_{\pm} = [a_{\mathbf{k}}, b_{\mathbf{p}}]_{\pm} = 0$$

Equations of motion (example)

$$E = p^2 + m^2$$

Relativistic scalar field:
Nonlinear Klein-Gordon equation

$$-\frac{\partial^2}{\partial t^2}\phi = -\nabla^2\phi + U(\phi)$$

$$U = -\frac{\partial V}{\partial \phi^*}$$

Relativistic fields:

- Representations of Poincare group (inhomogeneous Lorentz group)
- Classified by spin.

Spin 0: Scalar field -- Pi meson
½: Spinor field – Electron, nucleon, quark
1: Vector field – EM field, gauge fields
2: Tensor field -- Gravitational wave <- Not yet observed

Spin & statistics:
- integer fields obey Bose statistics,
- half-integer fields obey Fermi statistics.

QED -- quantum electrodynamics

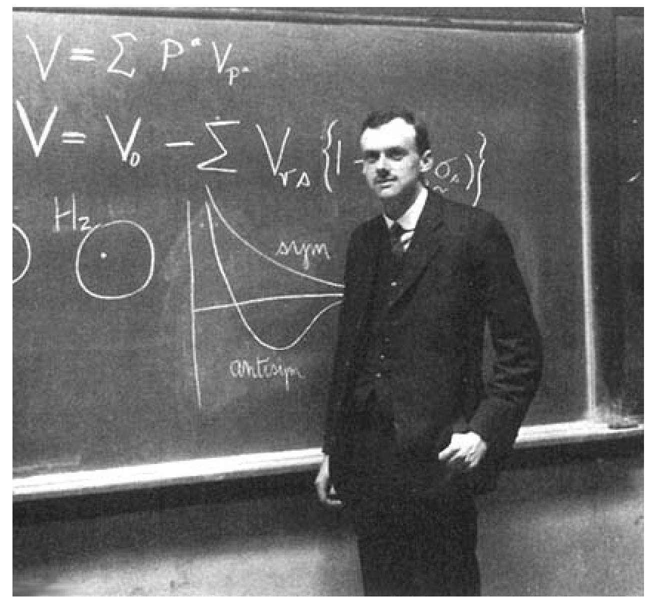
- Quantum field theory of interacting photons and electrons
- The most successful theory in physics
- Virtual processes → vacuum fluctuations
- Cutoff → renormalization

Experimental manifestations of vacuum fluctuations

- Lamb shift
- Electron anomalous magnetic moment
- Casimir effect

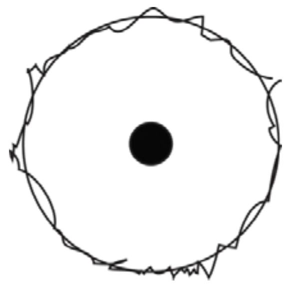
Hans A. Bethe 1906-2005 | Julian S. Schwinger 1918-1994 | Victor F. Weisskopf 1908-2002

Freeman J. Dyson 1923-



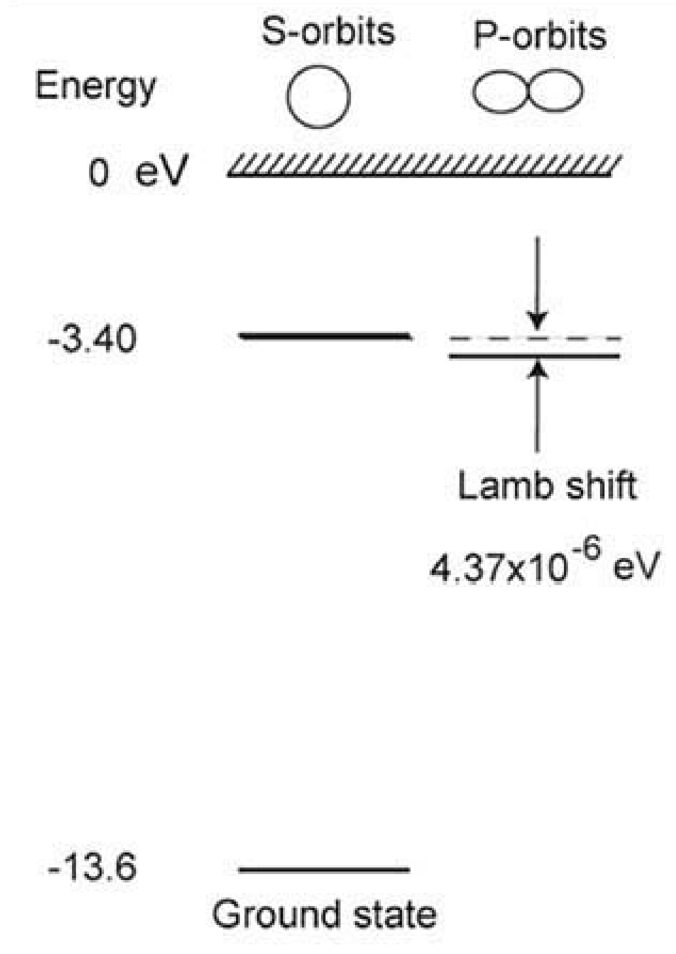
P.A.M. Dirac (1902-1984)

The Lamb shift in H atom



Electron orbit in H atom, in
Fluctuating vacuum EM field

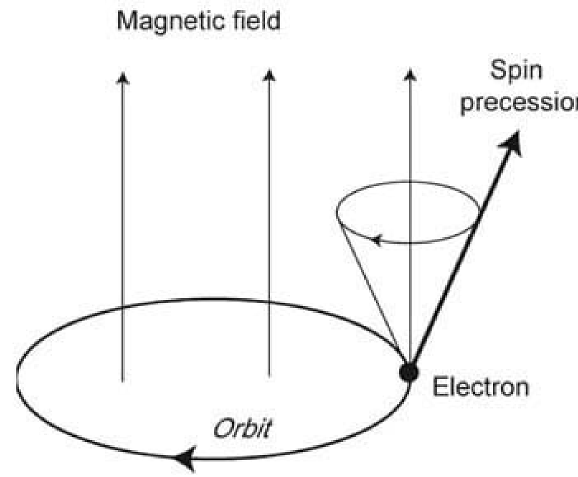
Willis E. Lamb
1913-



Electron anomalous magnetic moment

$$\tfrac{1}{2}g_{\text{expt}} = 1.00115965218085(76)$$

Polykarp Kusch
1911-1993



$$\begin{aligned}\tfrac{1}{2}g_{\text{theory}} &= 1 \\ &+(\alpha/2\pi) \\ &-0.32848(\alpha/\pi)^2 \\ &+(1.195 \pm 0.026)(\alpha/\pi)^3 \\ &-(1.7283(35))(\alpha/\pi)^4 + (\text{Non-QED})\end{aligned}$$

(a) 1928 (Dirac equation)

(b) 1949 (1 diagram)

(c) 1958 (18 diagrams)

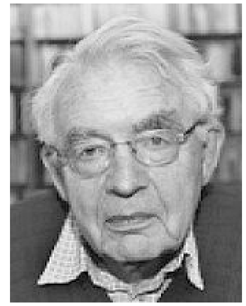
(d) 1974 (72 diagrams)

(e) 2006 (891 diagrams)

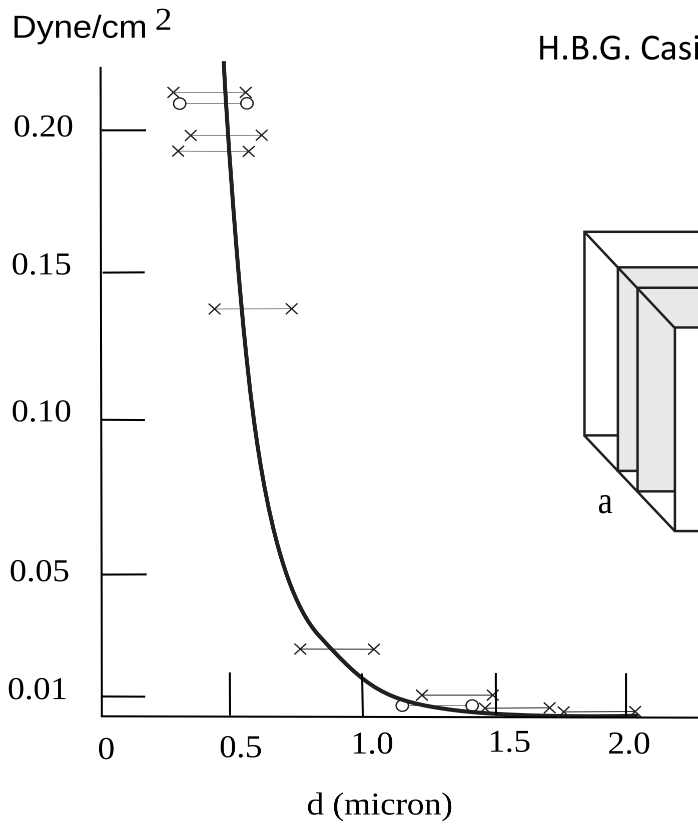
$$\frac{1}{\alpha} = 137.035999710(96)$$

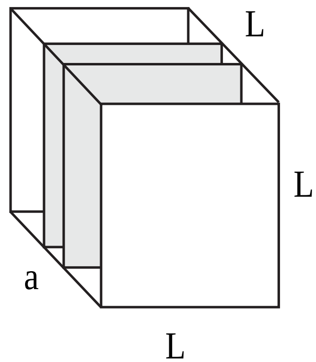
Casimir effect

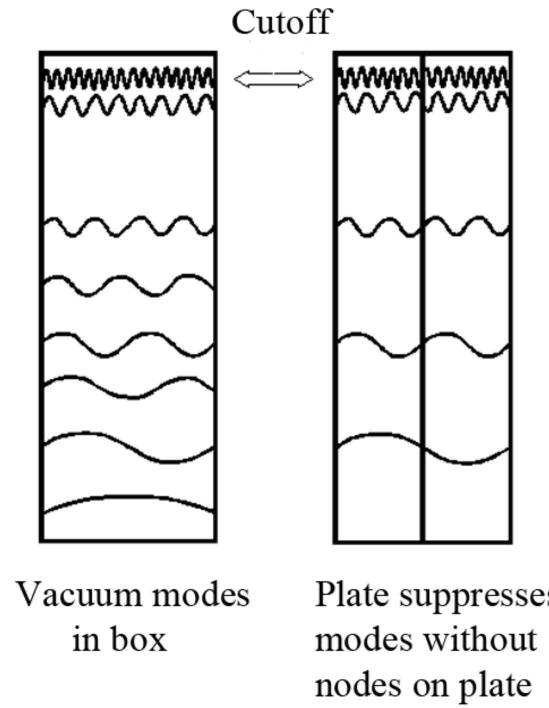
Two metal plates in vacuum attract each other.



H.B.G. Casimir (1909-2000)



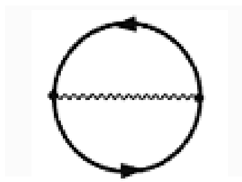




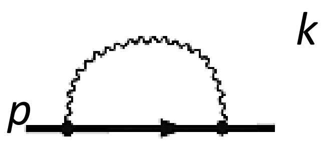
- Suppression of modes lowers vacuum energy.
- Mode-counting requires cutoff.

Cutoff and renormalization

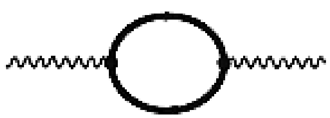
Virtual processes in Feynman graphs



Pure vacuum process



Electron self-energy δm



Photon self-energy – vacuum polarization

Example: electron self-energy

$$\delta m \sim e_0^2 \int_{|k|<\Lambda} \frac{d^4 k}{(p-k)^2 k^2} \sim e_0^2 \ln \Lambda$$

$$m = m_0 + \delta m$$

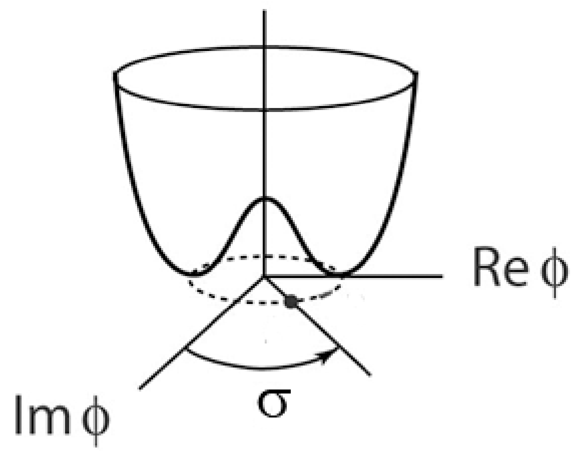
Take *m* from experiments.

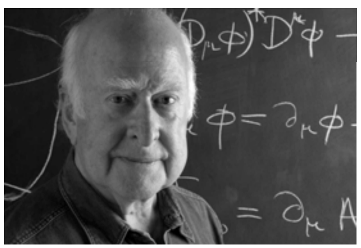
Vacuum scalar field – spontaneous symmetry breaking

Jeffrey R. Goldstone
(1933-)

"Higgs mechanism" generates mass in the standard model, through spontaneous breaking of local gauge invariance.

$$\langle \phi \rangle \neq 0$$
$$\langle \phi \rangle = F e^{i\sigma}$$





Peter Higgs (1929-)

$$\phi = \langle \phi \rangle + \eta$$
$\langle \phi \rangle =$ classical field

$\eta =$ quantum field

Spatial variation of the phase leads to superfluidity.

What is a superfluid?

A macroscopic quantum system with a coherent quantum phase.

- superconductor
- liquid helium
- cold trapped atoms in Bose-Einstein condensation (BEC)

A complex scalar field can describe a superfluid as a phenomenlogical “order parameter”, because it can have a coherent phase field over space.

$$\phi = F e^{i\sigma}$$

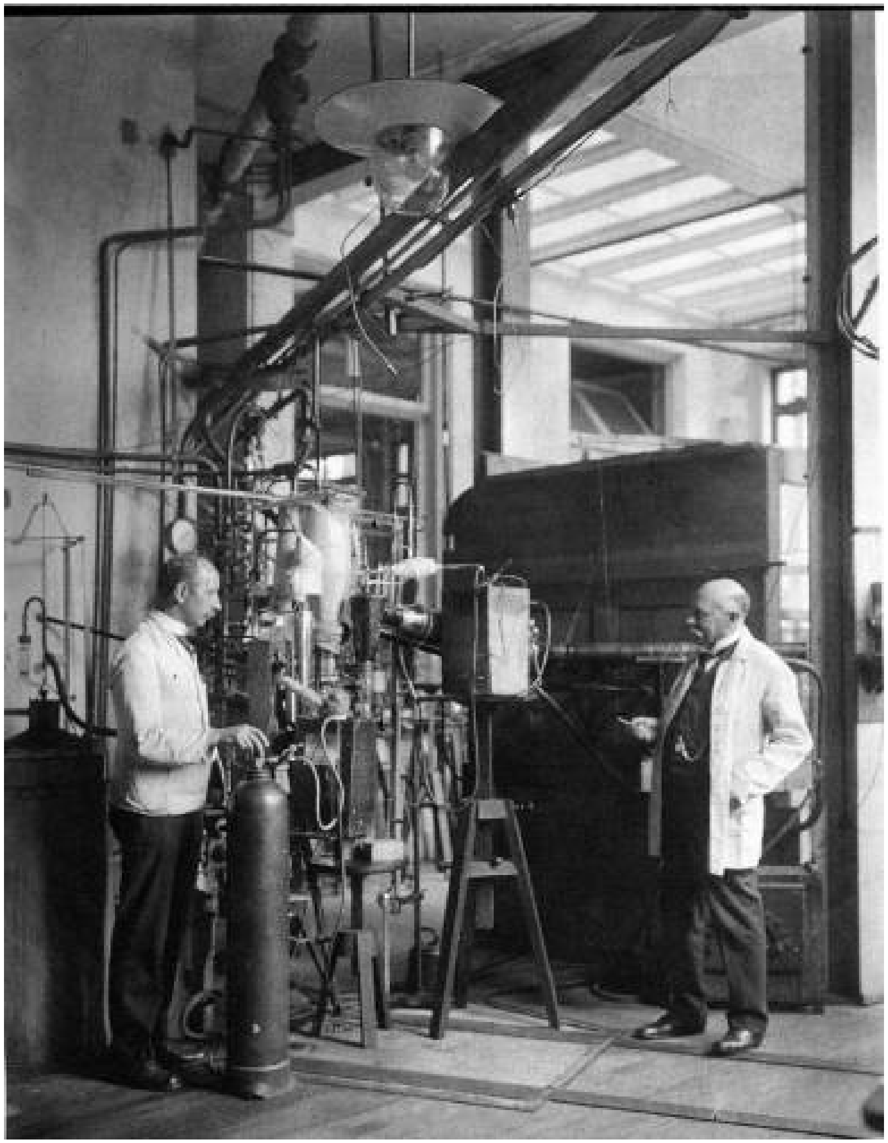
$$v = \kappa \nabla \sigma \quad \text{(Superfluid velocity)}$$

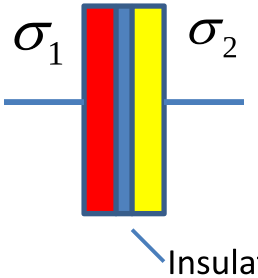
Superconductivity

Kamerlingh Onnes (1853-1926)

- Liquified helium (1908)
- Discovered superconductivity (1911)

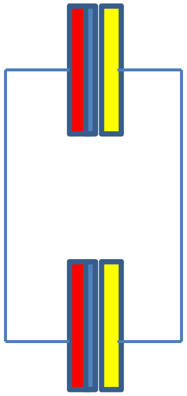
after spending weeks, with his assistant, in unsuccessful attempt to fix a “short circuit”.





Josephson junction

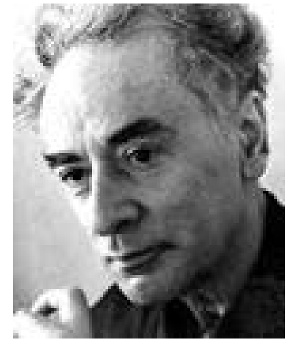
Sandwich of two superconductors, with different phases.



SQUID

Ginsburg-Landau theory of superconductivity

$\langle \phi \rangle =$ order parameter.
NLSE with magnetic field.
Spontaneous symm. breaking generates photon mass.
Meissner efffect: penetration depth $=$ (photon mass)$^{-1}$



Lev Landau (1908-1968)

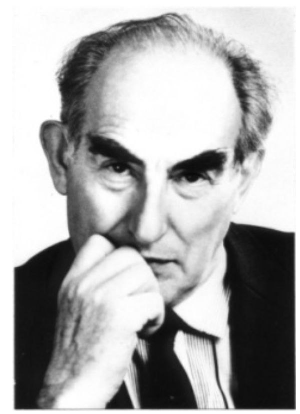
BCS : microscopic theory

Order parameter = condensate wave function
of Cooper pairs In Bose-Einstein condensation

John Bardeen
(1908-1991)

Leon N. Cooper
(1930-)

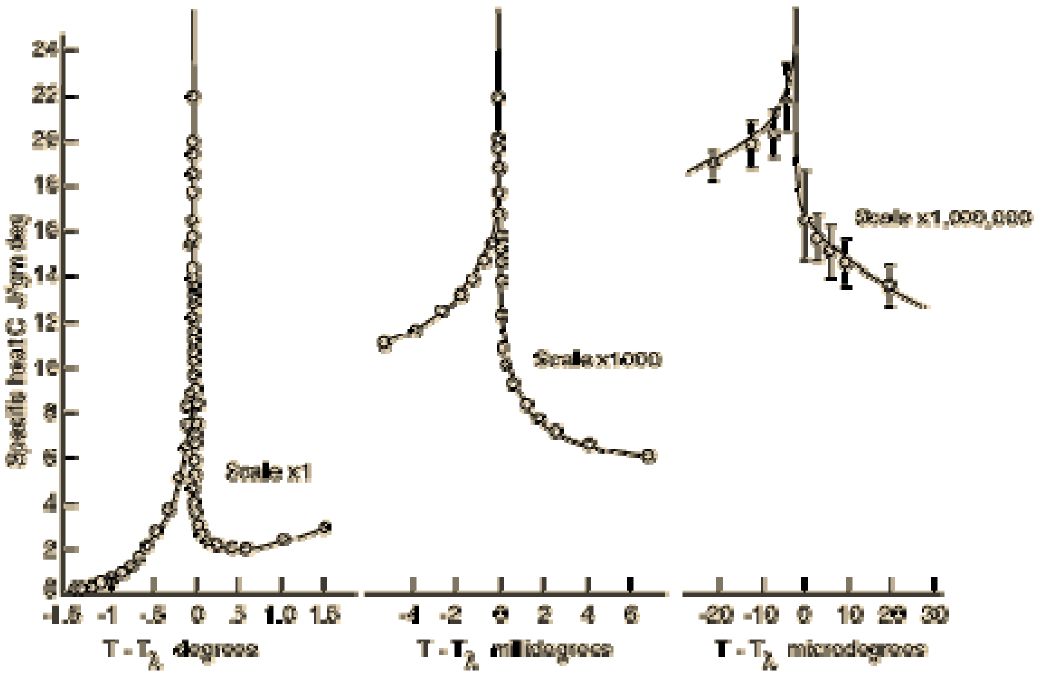
J. Robert Schrieffer
(1931-)



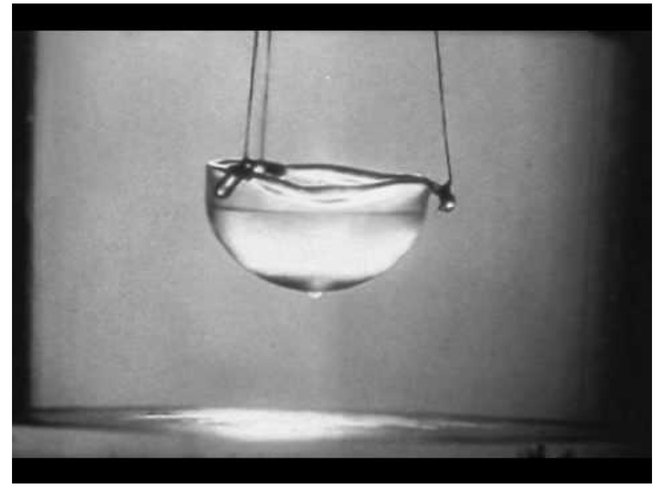
Vitaly Ginzburg (1916-2009)

Liquid helium

- Liquifaction at 3 K (-269 C)
- Superfluid phase at 2.18 K

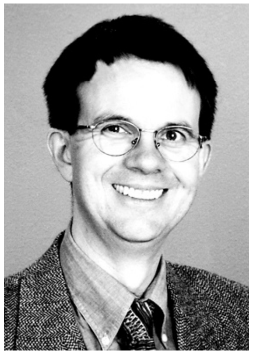


The lambda point: singular specific heat marks transition to superfluid -- Bose-Einstein condensation.

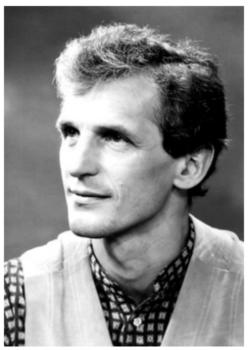


Superfluid cannot be contained.

Bose-Einstein condensation in atomic gases

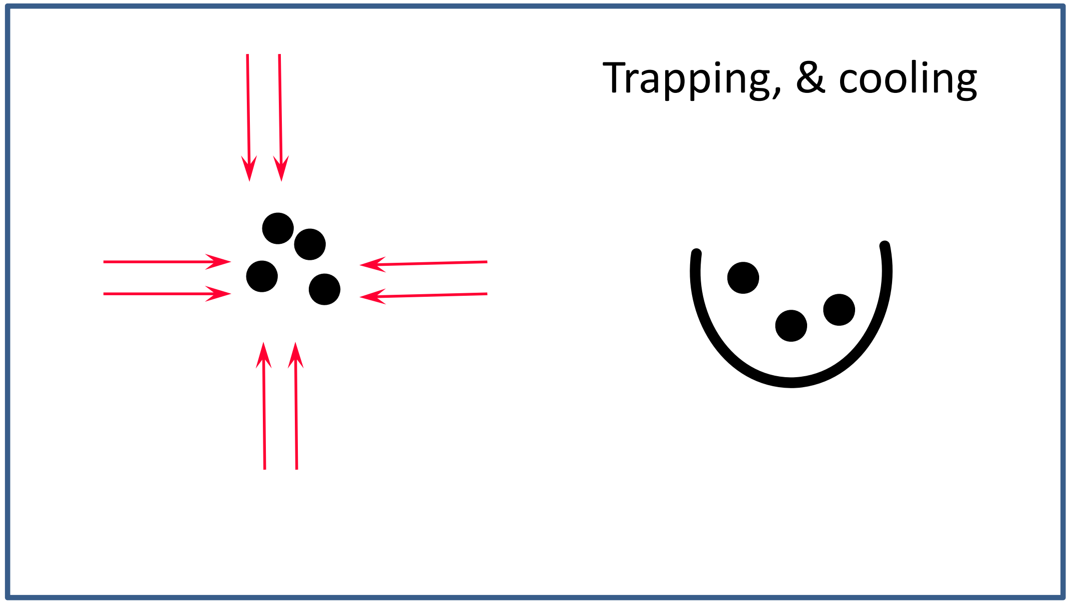


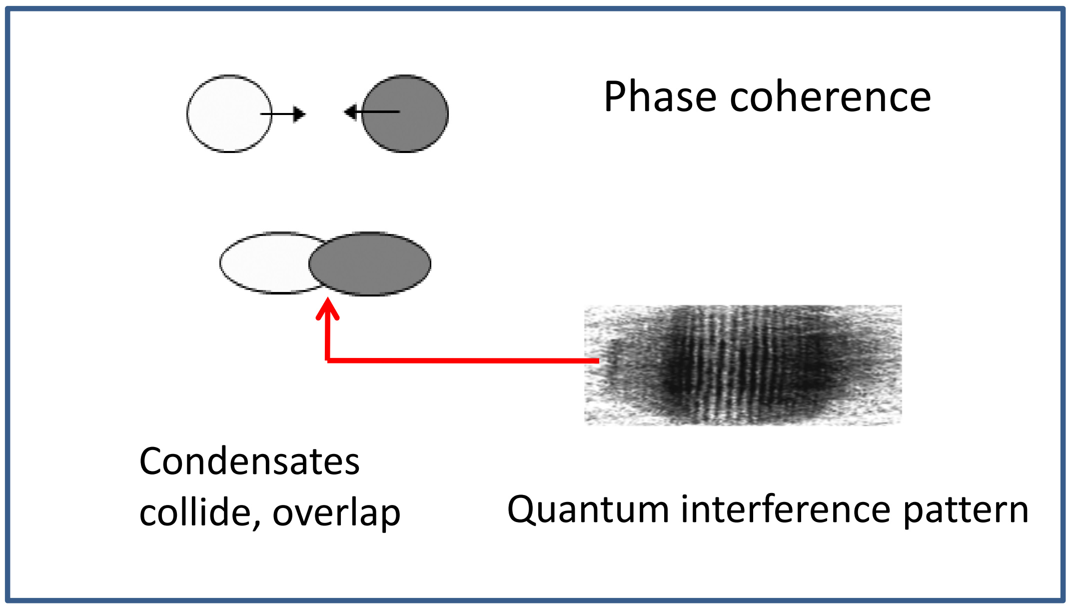
Eric A. Cornell (1961-)



Wolfgang Ketterle (1957-)

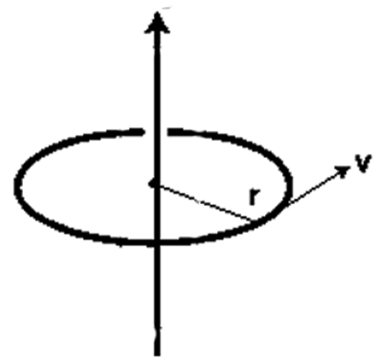
Carl E. Wieman (1951-)





Quantized vortex line

Vortex line



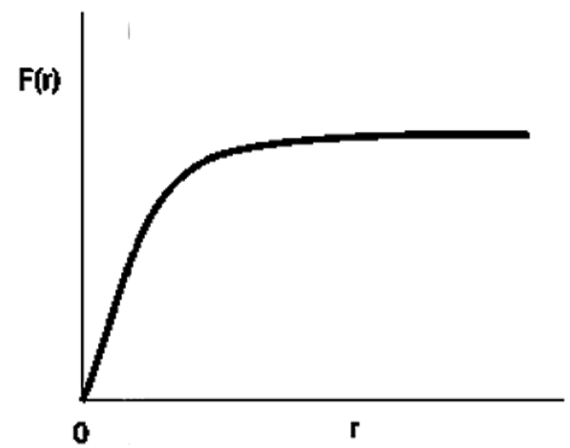
$$\phi = Fe^{i\sigma}$$

$$v = \kappa \nabla \sigma$$ (Superfluid velocity)

$$\oint v \cdot ds = \kappa \oint \nabla \sigma \cdot ds = 2\pi\kappa n$$

$$2\pi r v = 2\pi\kappa n$$

$$v = \frac{n\kappa}{r}$$



Vortex lines made visible

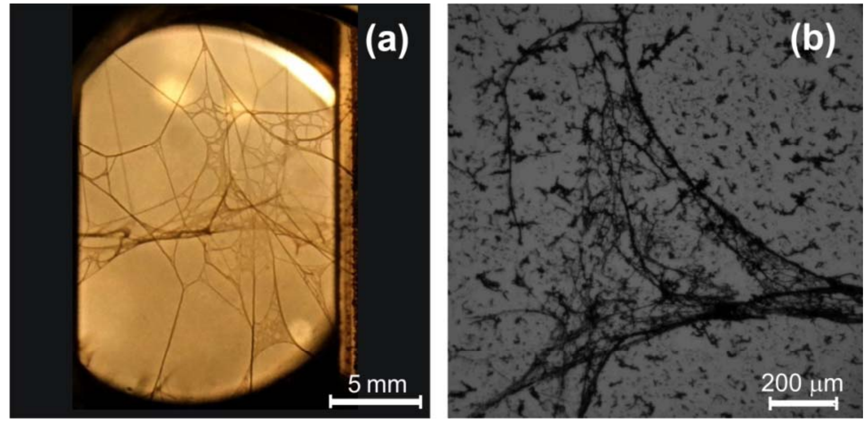
Metallic powder sticks to surface of vortex core in liquid helium

(University of Fribourg)

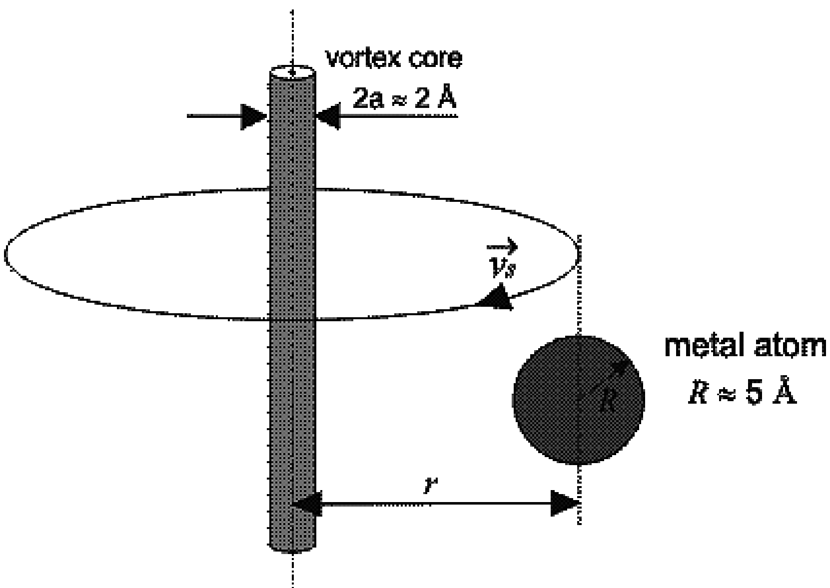
http://physics.unifr.ch/page/178/

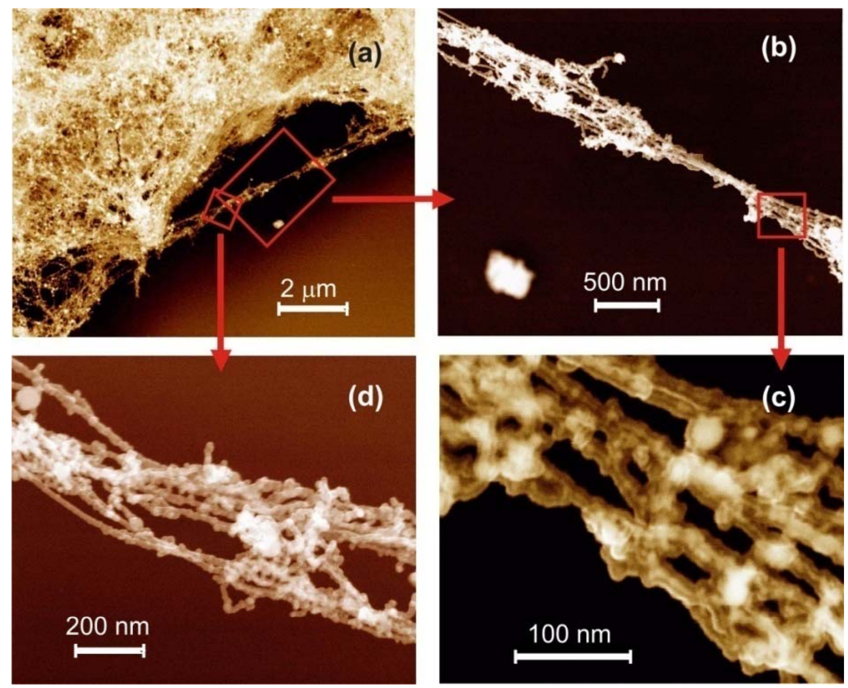
(V. Lebedev et al, J. Low Temp Phys.

DOI 10.1007/s10909-011-0384-7)



(a) Copper (b) gold

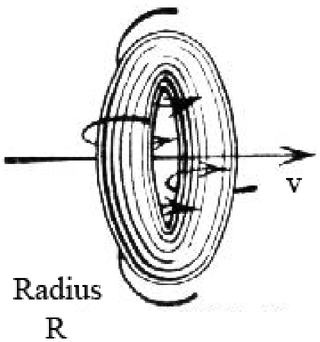


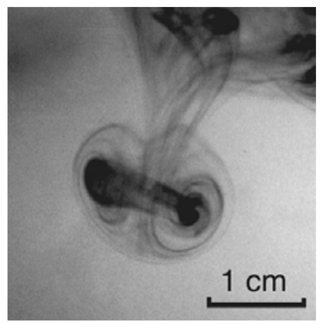


Under electron microscope

Vortex dynamics

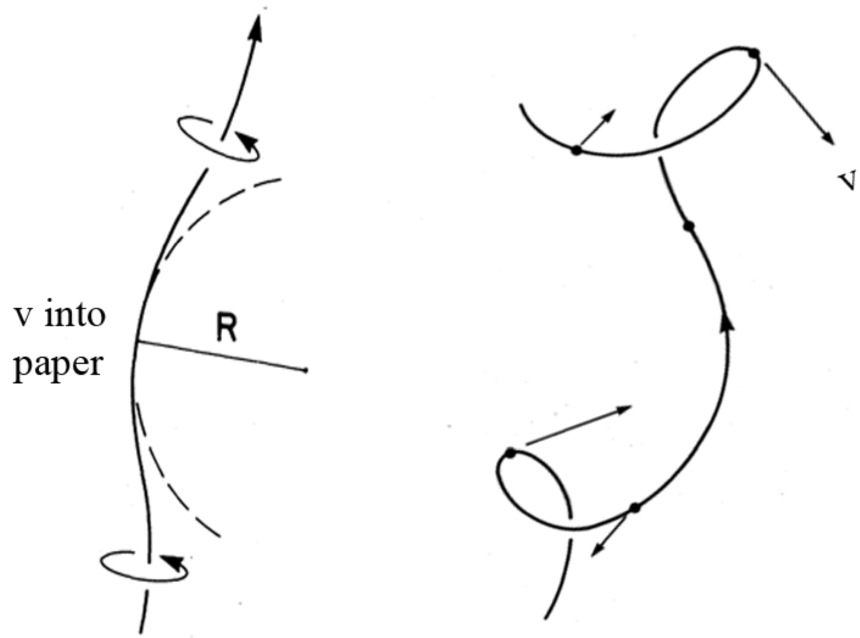
Elementary structure is vortex ring





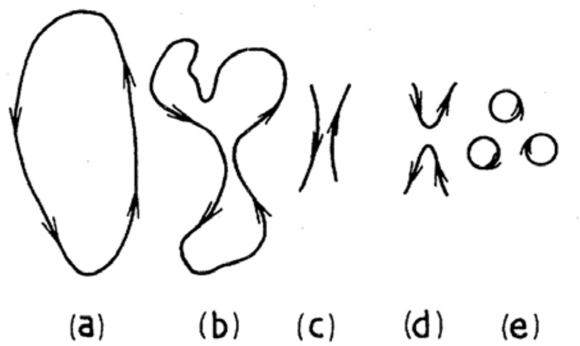
$$v = \frac{1}{4\pi R} \ln \frac{R}{R_0}$$

Self-induced vortex motion

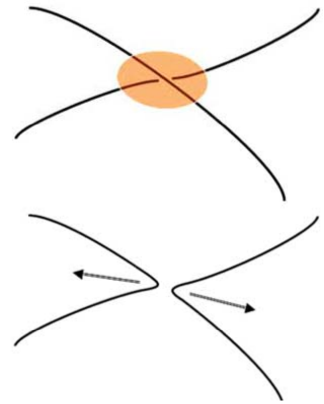


The smaller the radius of curvature R, the faster it moves normal to R.

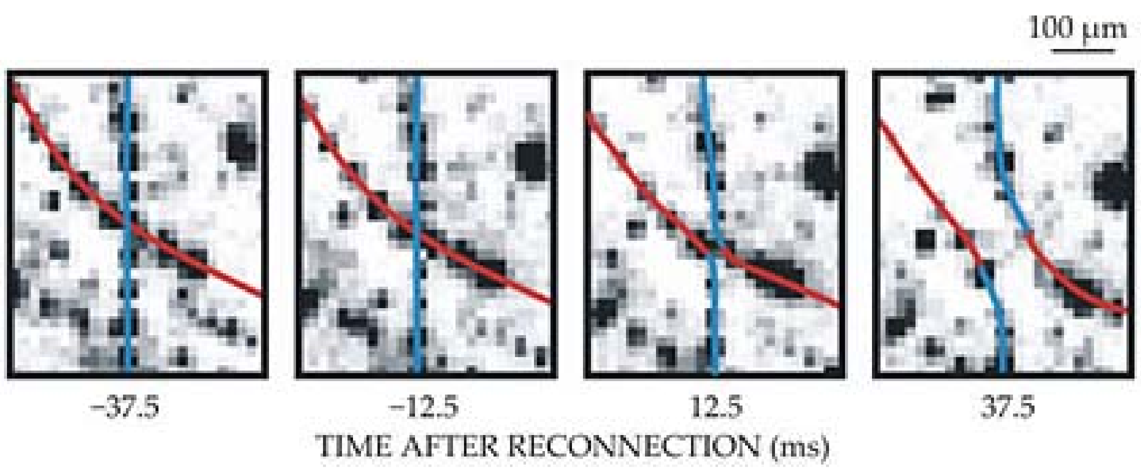
Vortex reconnection



Feynman's sketch

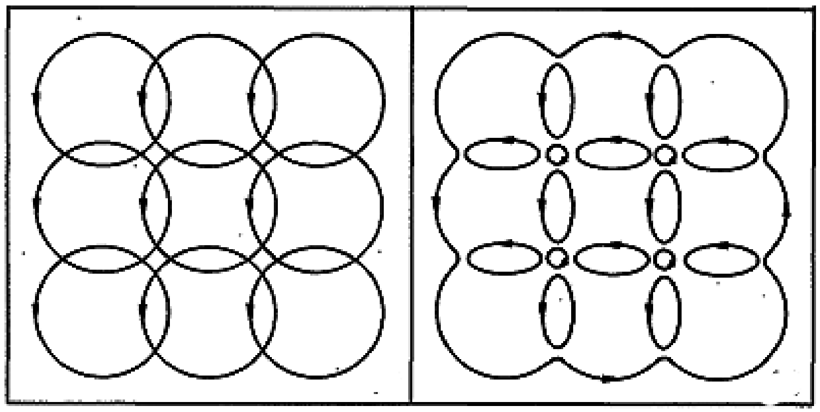


Signature: cusps spring away from each other at very high speed (due to small radii), creating two jets of energy.

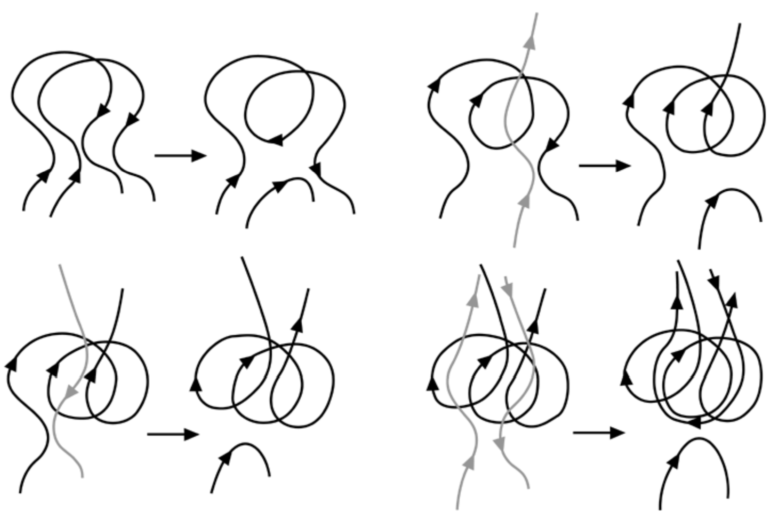


Observed vortex reconnection in liquid helium-- a millisecond event.
D. Lathrop, *Physics Today*, 3 June, 2010.

Change of topology due to reconnections

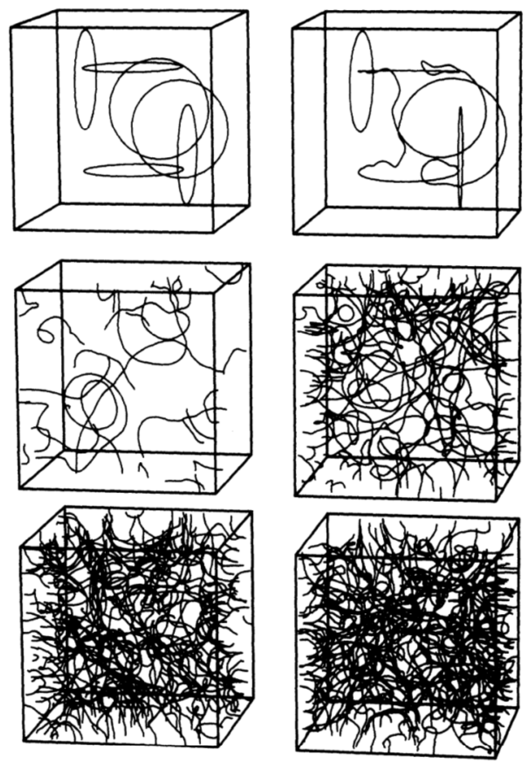


Magnetic reconnections in sun’s corona are responsible for solar flares.



Simulation of quantum turbulence [K.W. Schwarz, Phys. Rev. **B 38**, 2398 (1988)]

Fractal dimension = 1.6 [D. Kivotides, C.F. Barenghi, and D.C. Samuels. Phys. Rev. Lett. **87**, 155301 (2001).]



Number of reconnections:

0 3

18 844

1128 14781

ℓ = vortex tube density (length per unit spatial volume)

$$\dot{\ell} = A\ell^{3/2} - B\ell^{2}$$ (Vinen's equation)

Growth Decay