A Superfluid Universe

Lecture 3
The big bang & quantum turbulence

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Lecture 3.

- Scale change & renormalization
  - RG trajectories, fixed points
  - Halpern-Huang scalar field
- Cosmological equations
- Planck scale and nuclear scale
- Quantum turbulence – inflation era
- Dark energy
- Dark mass, galactic voids, and other phenomena

Big bang and renormalization

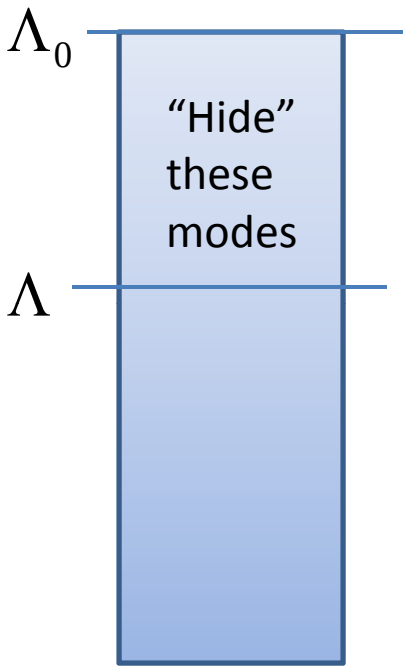
- Big bang is characterized by rapid change of length scale.
- Therefore, the cutoff of quantum fields change rapidly.
- With change of cutoff, interactions undergo renormalization.

$$\mathcal{L} = \partial\phi^* \partial\phi - V(\phi^*\phi)$$

$$V(\phi^*\phi) = \lambda_2 \phi^*\phi + \lambda_4 \left(\phi^*\phi\right)^2 + \lambda_6 \left(\phi^*\phi\right)^3 + \cdots$$

All the coupling constants change rapidly during the big bang.

In quantum field theory a cutoff is needed to suppress high-frequency virtual processes.



Momentum spectrum

The original cutoff defines the "bare" theory

Renormalized theory is a coarse-grained version, with lower effective cutoff.

- The cutoff is the only scale in the theory

$$\Lambda_0 \to \Lambda$$

- Scale change

- All coupling constants undergo **renormalization** to preserve the identity of the theory.

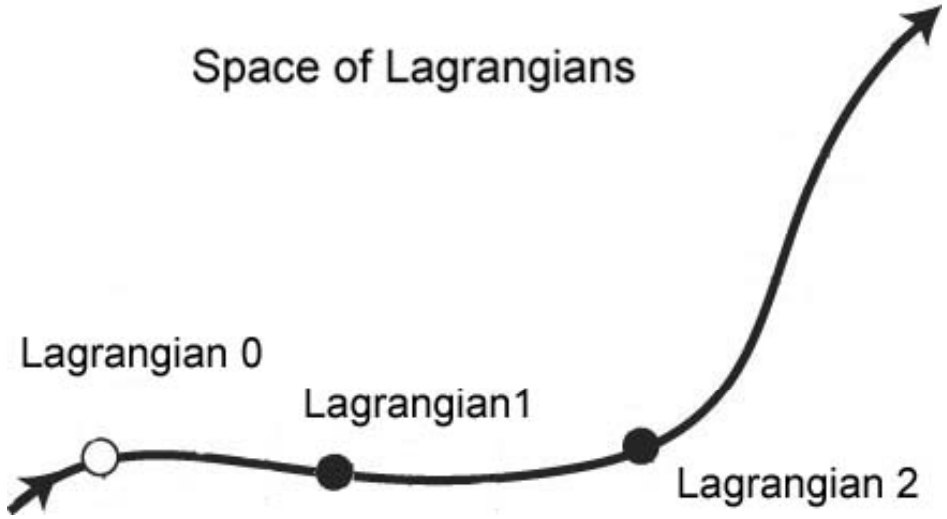
- The "appearance" of the theory is changed, but not the identity.

Scale changes form a group. Operations of renormalization form a representation called the renormalization group (RG).

RG trajectory

$$\mathcal{L} = \partial\phi^* \partial\phi - V\left(\phi^*\phi, \Lambda\right)$$

As Λ changes, $V\left(\phi^*\phi, \Lambda\right)$ traces out an RG trajectory in the space of Lagrangians

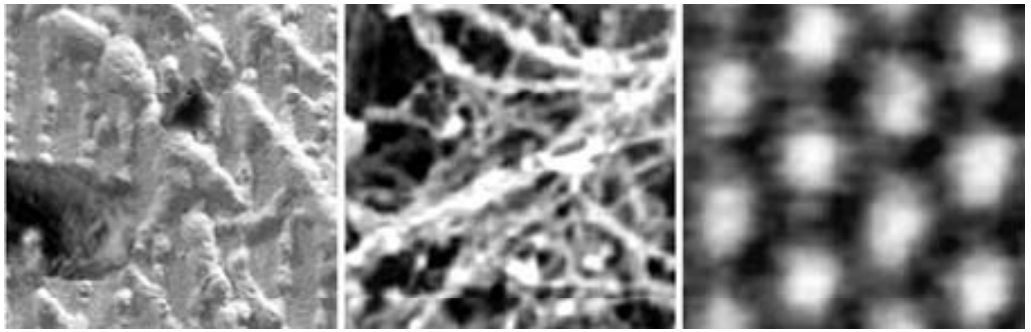


Kenneth G. Wilson
1936-

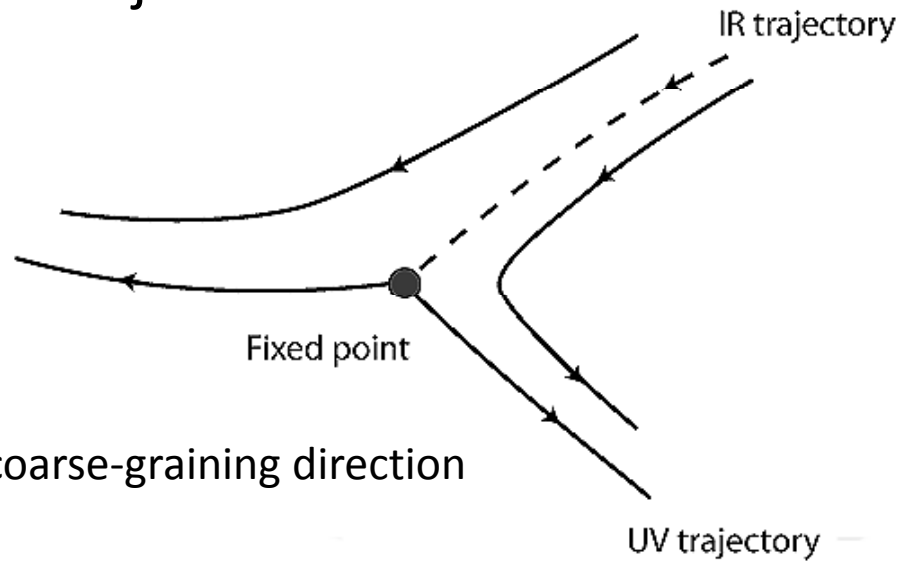
→ coarse-graining direction

Renormalization

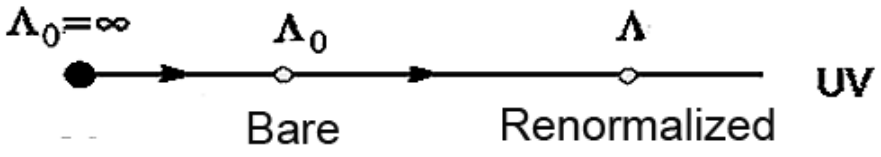
Appearances change under scale change, but not intrinsic identity.

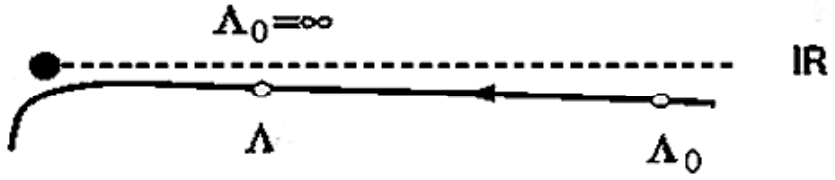


RG trajectories



→ coarse-graining direction

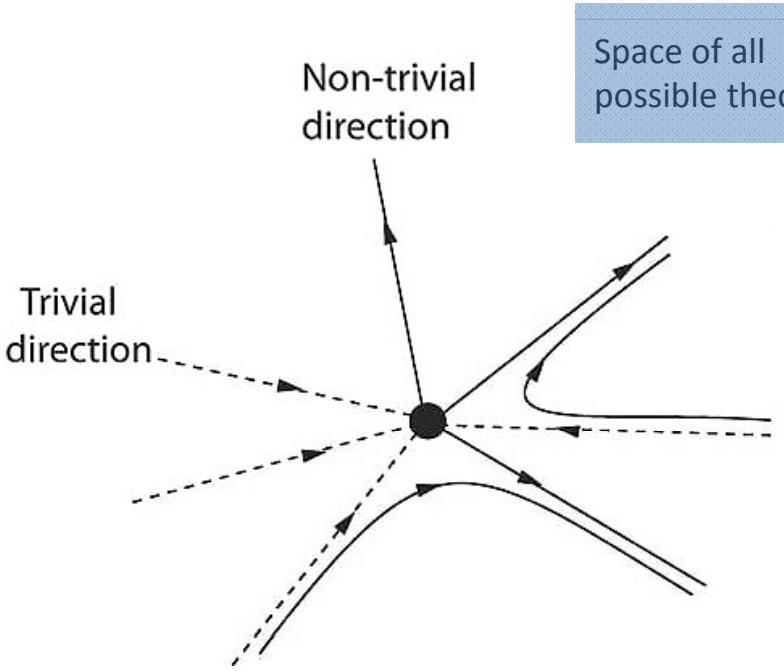




- Fixed point
- UV trajectory
  Asymptotic freedom
  QCD
- IR trajectory
  Non-free
  QED, ϕ^4
- Gaussian fixed point:
  Free massless scalar field

The Creation

- At big bang, universe at **Gaussian fixed point:** $\Lambda = \infty\,,\ \mathrm{V} \equiv 0$
- Scalar field gets on RG trajectory along particular direction (at random?)
- Direction corresponds particular form of V.



Outgoing trajectory --- Asymptotic freedom
Potential grows to spawn a possible universe.

Ingoing trajectory --- Triviality
Universe never left the fixed point.

Radius of universe gives field-theory cutoff:

$$\Lambda = \frac{\hbar}{a}$$

All asymptotically free potentials are **Halpern-Huang** potentials:

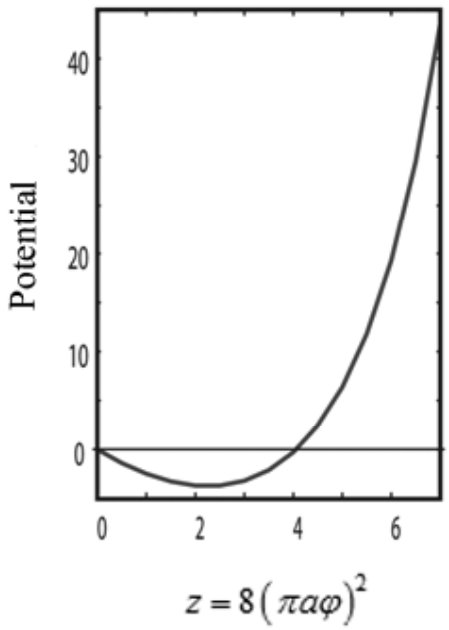
$$V = c\Lambda^{4-b}\left[M\left(-2+\frac{b}{2},\frac{N}{2},z\right)-1\right] \quad (0<b<2)$$

$$z = \frac{8\pi^2|\phi|^2}{\Lambda^2}$$

M = Kummer function
N = No. of field components

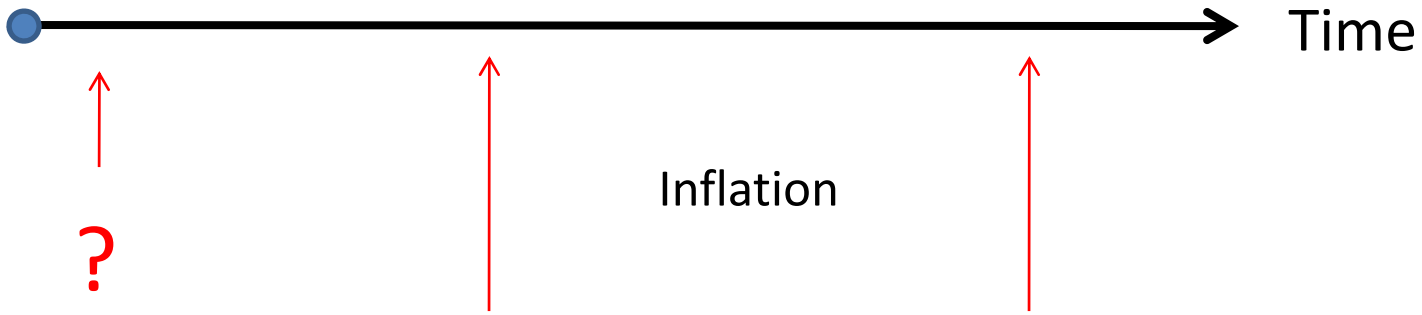
For large fields:

$$M(p,q,z) \approx \Gamma(q)\Gamma^{-1}(p)z^{p-q}\exp z$$



The field theory is *D=4* generalization of the sine-Gordon theory in *D=2*.

The big bang



Sub-Planckian era

- Universe very hot
- Rapidly cools
- Bose-Einstein condensation

Model starts
$O(10^{-43}$ s)

- Robertson-Walker metric
- Uniform complex vacuum field, HH potential
- Homogeneous quantum turbulence

Matter created
$O(10^{-26}$ s)

- Model ceases to be valid.
- Density fluc. Important,
- Standard hot big bang model takes over, but with superfluidity.

Relevant scales

Planck scale: Planck energy = 10^{18} GeV
(Built into Einstein's equation)

Nuclear scale: Nuclear energy = 1 GeV
This scale emerges spontaneously in QCD,
through formation of nucleon bound state
("dimensional transmutation").

Example of dimensional transmutation:

2D Schrödinger equation

$$-\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\psi-\lambda\delta(x)\delta(y)=E\psi \qquad (\lambda>0)$$

λ is dimensionless.

There is no intrinsic scale.

But there always exists a bound state.

Its energy is an emergent scale.

Robertson-Walker metric:

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right)$$ (k = 0, 1, -1)

For HH complex scalar field:

$$\rho = \frac{1}{2}|\dot{\phi}|^2 + V$$

$$p = \frac{1}{2}|\dot{\phi}|^2 - V - \frac{a}{3}\frac{\partial V}{\partial a}$$ (Last term = trace anomaly)

FLRW equations of motion: $H = \dot{a}/a$

$$\dot{H} = \frac{k}{a^2} - (p + \rho)$$

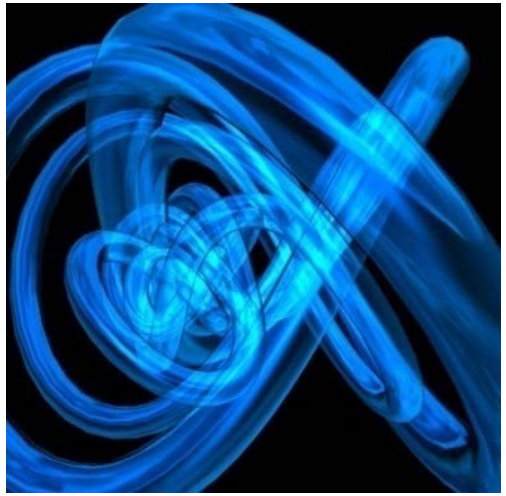
$$H^2 = -\frac{k}{a^2} + \frac{2}{3}\rho$$

$$\dot{\rho} = 3H(\rho + p)$$

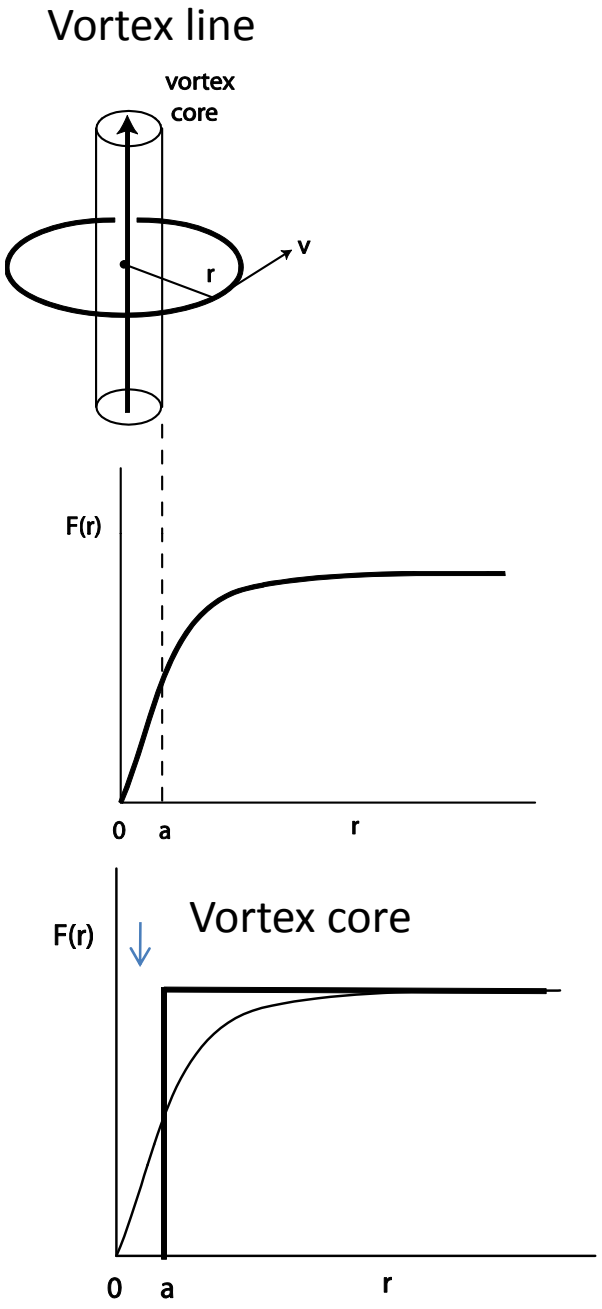
Plus scalar-field and matter equations

• In order to use the Robertson-Walker metric, the scalar field must be spatially uniform

• To describe vorticity in this metric, take vortex lines to be tubes, outside of which field is uniform.



Space becomes multiply-connected.



Variables

$a(t)$ = Radius of universe

$F(t)$ = Modulus of scalar field

$\ell(t)$ = Vortex tube density

$\rho(t)$ = Matter density

$E_{\mathrm{v}} = a^3 \varepsilon_0 \ell$ (Total vortex energy)

$E_{\mathrm{m}} = a^3 \rho$ (Total matter energy)

Dynamics:

$\dot{a}(t)$ from Einstein's equation with RW metric.

Source of gravity: $T^{\mu\nu}_{\mathrm{tot}} = T^{\mu\nu}_{F} + T^{\mu\nu}_{\ell} + T^{\mu\nu}_{\rho}$

$\dot{F}(t)$ from field equation.

} Planck scale

$\dot{\ell}(t)$ from Vinen's equation.

$\dot{\rho}(t)$ determined by conservation law $T^{\mu\nu}_{\mathrm{tot};\mu} = 0$.

} Nuclear scale

Cosmological equations: $(4\pi G = c = \hbar = 1)$ $H = \frac{\dot{a}}{a}$

Planck scale

$$\frac{dH}{dt} = \frac{k}{a^2} - 2\left(\frac{dF}{dt}\right)^2 + \frac{a}{3}\frac{\partial V}{\partial a} - \frac{1}{a^3}\left(E_{\mathrm{m}} + E_{\mathrm{v}}\right)$$

$$\frac{d^2F}{dt^2} = -3H\frac{dF}{dt} - \frac{\zeta_0 E_{\mathrm{v}}}{a^3}F - \frac{1}{2}\frac{\partial V}{\partial F}$$

Nuclear scale

$$\frac{dE_{\mathrm{v}}}{d\tau} = -E_{\mathrm{v}}^2 + \gamma E_{\mathrm{v}}^{3/2}$$

$$\frac{dE_{\mathrm{m}}}{d\tau} = \left\langle \frac{\zeta_0}{s_1}\frac{dF^2}{dt} \right\rangle E_{\mathrm{v}}$$

Essentially constant

- Rapid change
- Av. over t
- of order 10^{18}

Constraint:

$$H^2 + \frac{k}{a^2} - \frac{2}{3}\left(\dot{F}^2 + V + \frac{1+\zeta_0}{a^3}E_{\mathrm{v}} + \frac{1}{a^3}E_{\mathrm{m}}\right) = 0$$

The two sets decouple because

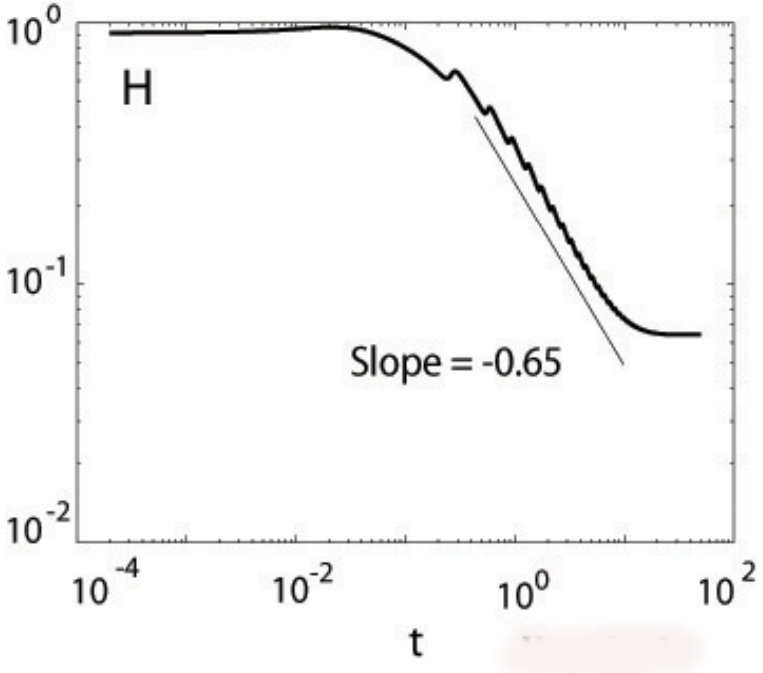
$$s_1 = \frac{\tau}{t} = \frac{\text{Planck time scale}}{\text{Nuclear time scale}} = \frac{\text{Nuclear energy scale}}{\text{Planck energy scale}} \sim 10^{-18}$$

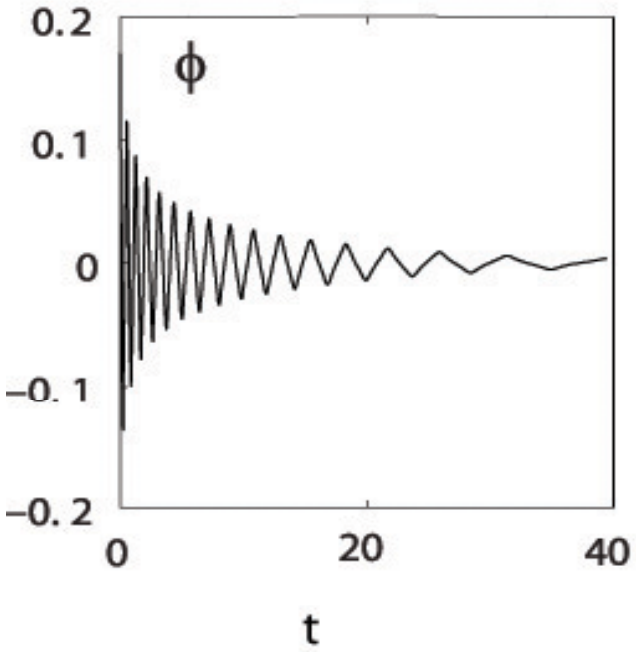
Numerical solutions

$$H \approx t^{-p}$$

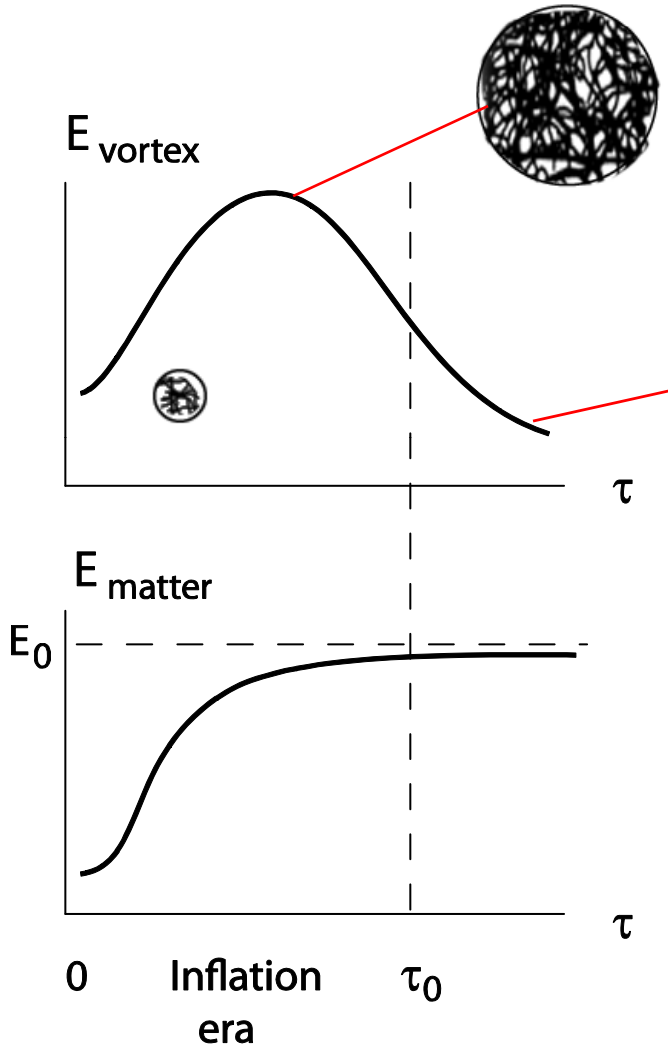
$$a \approx \exp\left(t^{1-p}\right)$$

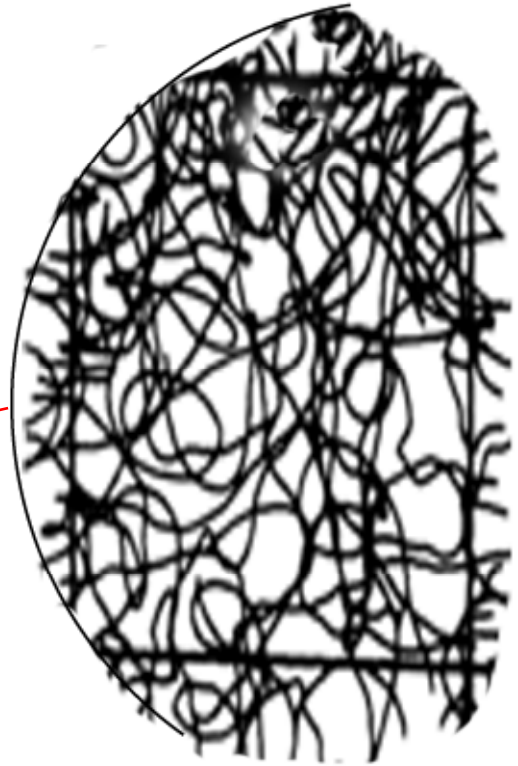
- Time-averaged asymptotic behavior: power law
- Gives dark energy without “fine-tuning” problem





Inflation era: quantum turbulence





Radius increases by factor 10^{27}
- in 10^{-26} seconds.
- Matter created = 10^{22} sun masses
- Eventually form galaxies outside of vortex cores.

At end of quantum turbulence, model goes over to standard hot big bang theory. But universe remains a superfluid with vorticity.

Inflation era = lifetime of quantum turbulence

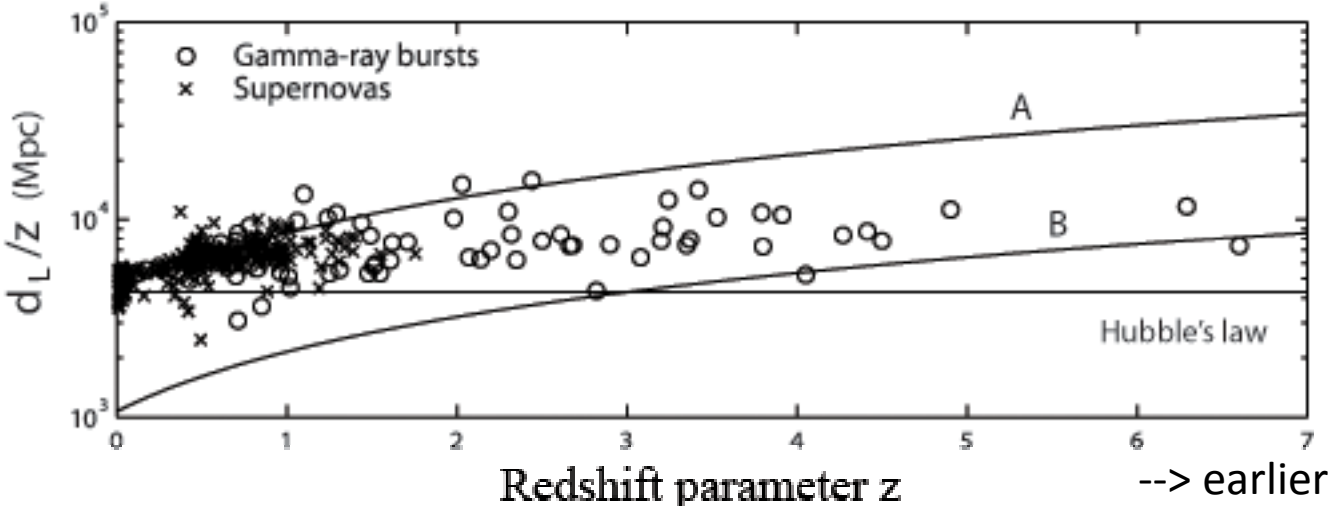
- Vortex tangle (quantum turbulence) grows and decays.
- All the matter needed for galaxy formation was created in the tangle.

Legacy

- After decay of quantum turbulence, standard hot big bang theory takes over.
- But the universe remains a superfluid.

Dark energy

Galactic redshift (d_L = luminosity distance)



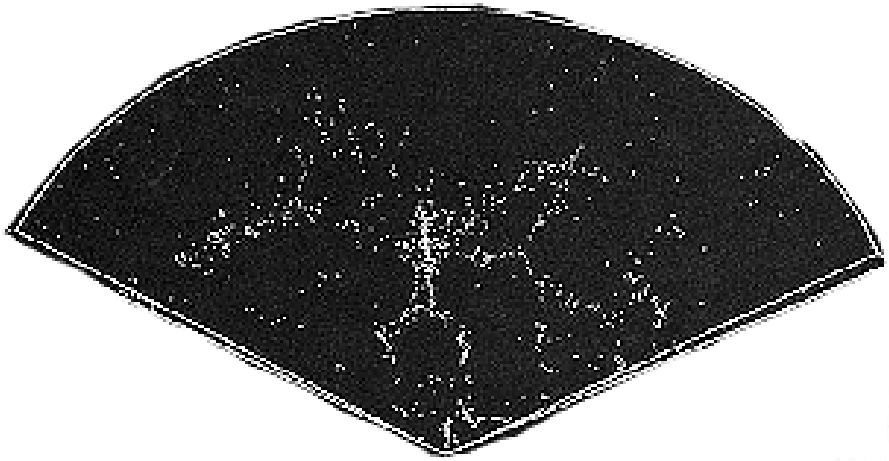
--> earlier epoch

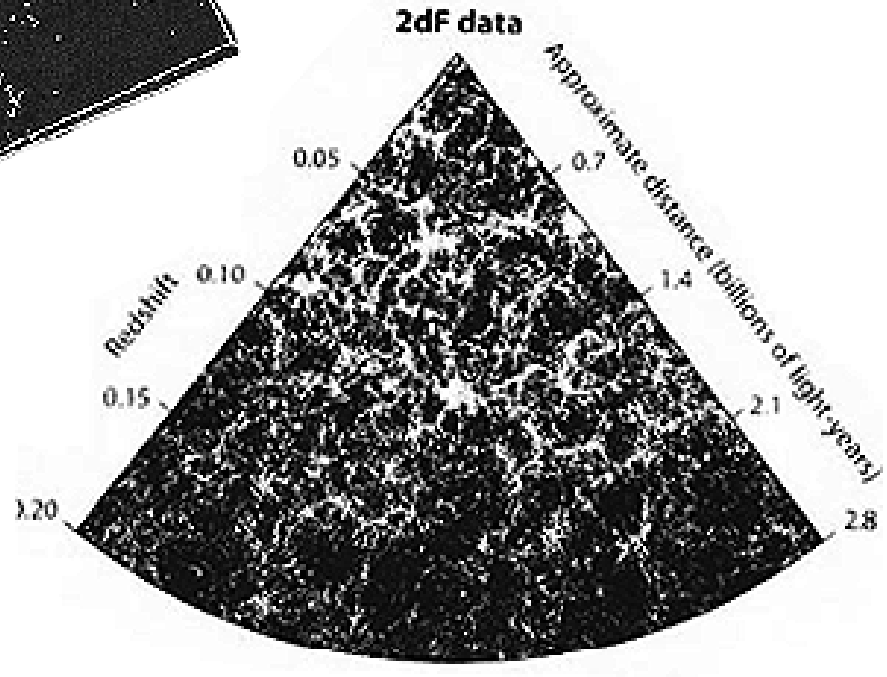
Hubble's law: horizontal line
Dark energy: accelerated expansion (points lie above line)

Theory: A,B.
Exponents *p* only affects vertical displacement.

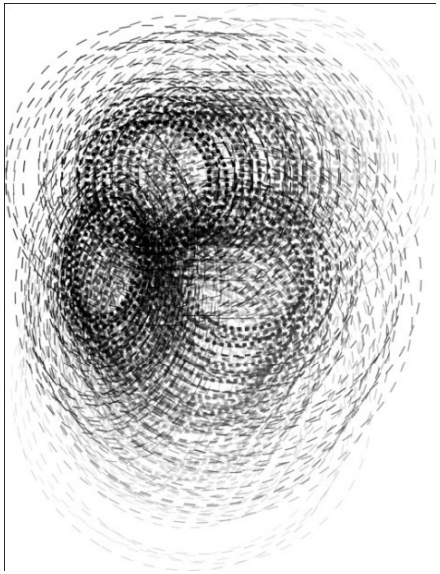
The “stick man”

Voids in galactic distribution



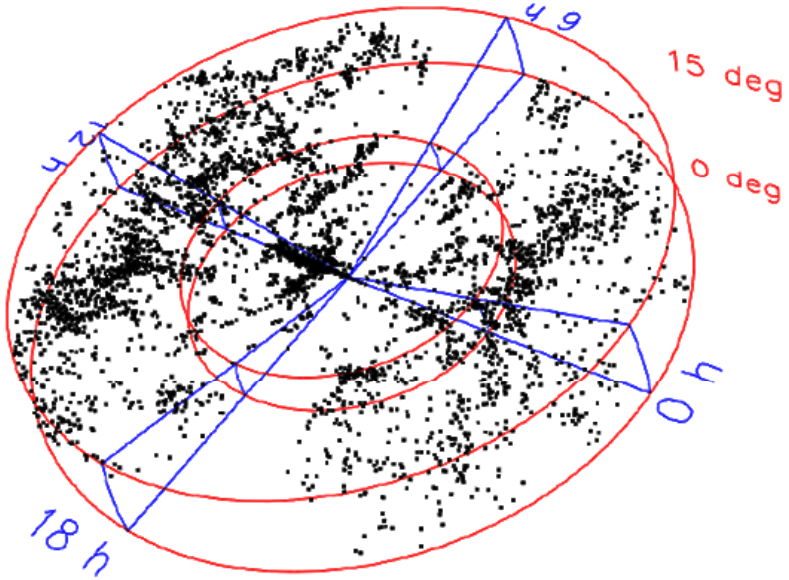


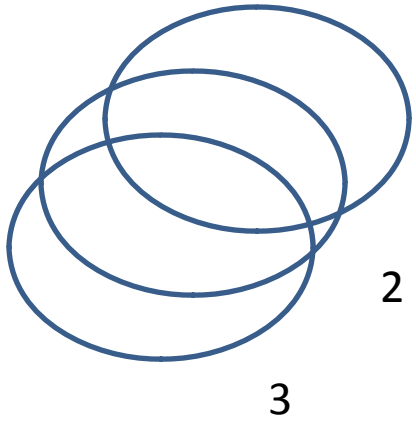
The “Great Wall”



“Stick man” simulated by three vortex cores.
Galaxies stick to surface of vortex tube.

- Slice cylinder of data into stacks.
- Compare voids in successive stacks.
- Do they form a tube?





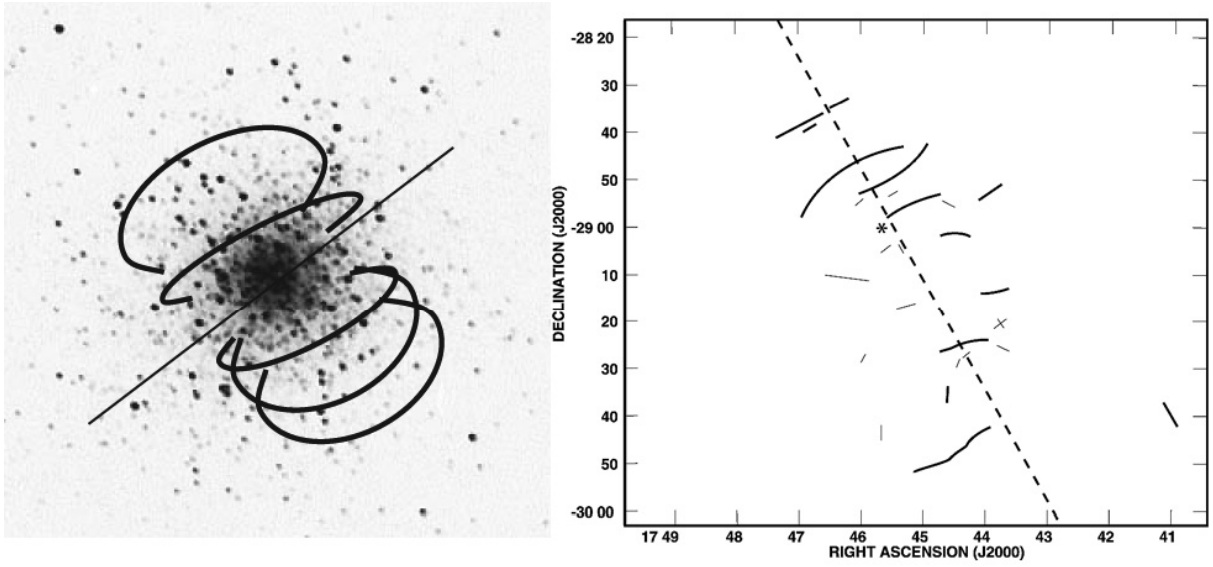
Dark mass and “non-thermal filaments”

• Superfluid can be pinned by a random potential

K. Huang ang H.-F. Meng, "Hard-sphere Bose gas in external random potentials", Phys. Rev. Lett., 69, 644 (1992).

• Galaxy or star cluster could drag the superfluid with it in rotation, acquire extra moment of inertia seen as dark mass.

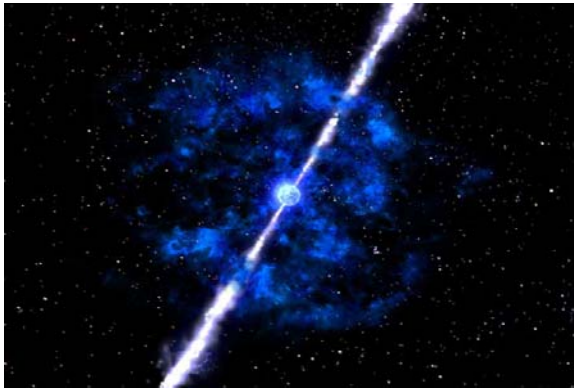
• Between the co-rotating superfluid and the background will be a boundary layer laced with vortex lines, manifested as the “non-thermal filaments”.



“Non-thermal filaments" observed near the center of the Milky Way

In later universe, reconnections of huge vortex tubes will be rare but spectacular.

Gamma ray burst



- A few events per galaxy per million years
- Lasting ms to minutes
- Energy output in 1 s

= Sun's output in entire life (billions of years).

Cosmic jet

Jet of matter 27 light years long