Effective boundary conditions for the Stokes fluid with thin coatings via homogenization

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In this report, we consider a mathematical problem arising from the fluids by applying a thin coat of special area to a body to be protected. The body is assumed to be a 2-Dimensional smooth and bounded region denoted by Ω_1. The thin coat occupies the region

$$\Omega_2 := \{x \in R^2 \setminus \Omega_1 | d(x) < \delta\} \quad \text{where} \quad d(x) := dist(x, \bar{\Omega}_1) = \min_{y \in \bar{\Omega}_1} |x - y|, \quad (1)$$

where δ is the thickness of Ω_2 satisfying $\delta \ll 1$.

We study the Stokes fluid in the coated body

$$\begin{cases} \dfrac{\partial u^\delta}{\partial t} - \text{div}(\mathcal{A}\nabla u^\delta) + \nabla p^\delta = 0, \quad \text{in} \;\; \Omega, \\ \text{div} u^\delta = 0 \quad \text{in} \;\; \Omega, \\ u^\delta|_{t=0} = u_0^\delta(x), \quad u^\delta|_{\partial\Omega} = 0, \qquad \text{for} \;\; t > 0, \end{cases} \quad (2)$$

where u^δ and p^δ are the velocity and the pressure of the fluid respectively. $\mathcal{A}$ is a 2×2 matrix, which is positive and symmetry. It is assumed that the interior body Ω_1 is homogeneous , namely,

$$A(x) = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \forall x \in \Omega_1. \quad (3)$$

Within the coating Ω_2, we assume the tensor $\mathcal{A}$ takes the form

$$\mathcal{A}(x)\nu = \sigma\nu, \quad \mathcal{A}(x)\tau = \mu\tau, \quad \forall x \in \Omega_2, \quad (4)$$

where σ and μ are positive constants, $\nu = \nabla d(x)$ is the unit normal vector and $\tau \perp \nu$ is a unit tangent vector.

Our goal is to find the behavior of u^δ, p^δ as δ tends to zero. We also want to find out the boundary conditions of the effective model.