Unramified endomorphisms of algebraic surfaces

Abstract: The Jacobian conjecture is still open and is short of any definitive approach. In the lecture, two approaches will be explained.

(1) Given an algebraic surface X defined over the complex field C together with an unramified endomorphism $\phi : X \to X$, we ask if ϕ is an automorphism. This generalization has positive or negative answers depending on the properties of X. In particular, it is an automorphism if X has a singular point which is not a quotient singularity.

(2) In the case of the affine plane, if (f, g) is a Jacobian pair, the conjecture is reduced to the case where $C[x, y]$ is the normalization R of $C[x, f, g]$ in its quotient field and R is unramified over $C[x, f, g]$. Analysis of unramifiedness of R over $C[x, f, g]$ involves subtle and difficult problems.

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