Algebraic Manifolds with Vanishing Hodge Cohomology

Jing Zhang, SUNY at Albany, USA

Let Y be a complex manifold with $H^i(Y, \Omega_Y^j) = 0$ for all $j \geq 0$ and $i > 0$, then what is Y? Is Y Stein? This is a question raised by J.-P. Serre, which is still open even for a surface. Here Ω_Y^j is the sheaf of holomorphic $j-$forms. Assume that Y is an algebraic manifold of dimension $d > 1$ and X a smooth completion of Y such that the boundary $X - Y$ is support of an effective divisor D on X with normal crossings. We will investigate the properties of Y. For example, Y cannot have $d-1$ algebraically independent nonconstant regular functions ($\kappa(D, X) \neq d-1$). The interesting phenomenon is that $\kappa(D, X)$ can be even if the dimension of Y is even and $\kappa(D, X)$ can be odd if the dimension of Y is odd.