Dynamic Corporate Default Predictions – Spot and Forward-Intensity Approaches

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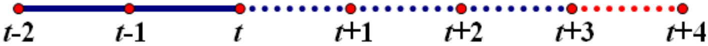
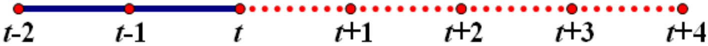
Outline

Questions of interest

- Single-obligor default prediction over different future periods (forward and cumulative)
  - Forward default probability



  - Cumulative default probability



- Portfolio credit analysis
  - Frequencies of defaults over some horizon
  - Exposure-weighted default distribution over some horizon

Literature Review

- Choosing between structural and reduced-form modeling approaches
- Discriminant analysis
    - Beaver (1966, 1968), Altman (1968), etc.
    - Model output: credit scores
- Binary response models: logit/probit regressions
    - Ohlson (1980), Zmijewski (1984), etc.
    - Model output: default probability in the next one period

- Campbell, *et al* (2008): logit models for different periods ahead

Literature Review (Cont'd)

- Recent development: duration analysis
  - Shumway (2001), Chava and Jarrow (2004), etc.
- The model of Duffie, Saita and Wang (2007):
  - Two Poisson processes (conditionally independent)
    1. Default/bankruptcy
    2. Other exit: merger and acquisition, etc.
  - Use spot intensities
    - Instantaneous rate of occurrence
    - Functions of the covariates (stochastic and deterministic)
  - Need to specify the time-series dynamics for the stochastic covariates

Literature Review (Cont'd)

- The model of Duan, Sun and Wang (2012):
  - Two Poisson processes (conditionally independent)
    1. Default/bankruptcy
    2. Other exit: merger and acquisition, etc.
  - Forward intensity
    - Instantaneous rate of occurrence
    - Functions of the covariates (stochastic and deterministic)
  - No need to specify the time-series dynamics for the stochastic covariates

Spot intensity model

- Let instantaneous default and other exit intensities for the *i*-th firm at time t be λ_{it} and ϕ_{it}, respectively.

- Define two stopping times
  τ_{Di}: default time of the *i*-th firm
  τ_{Ci}: combined exit time of the *i*-th firm

- The default probability over $[t, t+\tau]$ becomes
  $E_t\left(\int_t^{t+\tau} e^{-(\lambda_{is}+\phi_{is})(s-t)}\lambda_{is}ds\right)$.

Spot intensity model (Cont'd)

- Model λ_{it} and ϕ_{it} as functions of state variables available at time t.
- $\lambda_{it} \geq 0$ and $\phi_{it} \geq 0$.
- $X_{it} = (x_{it,1}, x_{it,2}, \cdots, x_{it,k})$: the set of the state variables

$$\begin{aligned}
\lambda_{it} &= \exp\{\lambda \exp[-\delta(t - t_B)] 1_{t>t_B} \\
&\quad + \alpha_0 + \alpha_1 x_{it,1} + \alpha_2 x_{it,2} + \cdots \alpha_k x_{it,k}\} \\
\phi_{it} &= \exp\left(\beta_0 + \beta_1 x_{it,1} + \beta_2 x_{it,2} + \cdots \beta_k x_{it,k}\right)
\end{aligned}$$

  where t_B is August 2008 (Note that the US government bailed out AIG in September 2008)
- Discretize the model for empirical implementation

The likelihood function

$$\mathscr{L}(\alpha, \beta; \tau_C, \tau_D, X) = \prod_{i=1}^{N} \prod_{t=0}^{T-1} \mathscr{L}_{i,t}(\alpha, \beta)$$

$$\begin{aligned}\mathscr{L}_{i,t}(\alpha, \beta) = & 1_{\{t_{0i} \leq t, \tau_{Ci} > t+\Delta t\}} P_t(\tau_{Ci} > t + \Delta t) \\ & + 1_{\{t_{0i} \leq t, \tau_{Di} = \tau_{Ci} = t+\Delta t\}} P_t(\tau_{Di} = \tau_{Ci} = t + \Delta t) \\ & + 1_{\{t_{0i} \leq t, \tau_{Di} \neq \tau_{Ci}, \tau_{Ci} = t+\Delta t\}} P_t(\tau_{Di} \neq \tau_{Ci} \& \tau_{Ci} = t + \Delta t) \\ & + 1_{\{t_{0i} > t\}} + 1_{\{\tau_{Ci} \leq t\}}\end{aligned}$$

- $P_t(\tau_{Ci} > t + \Delta t)$: probability of surviving both forms of exit over the next period
- $P_t(\tau_{Di} = \tau_{Ci} = t + \Delta t)$: probability that firm defaults in the next period
- $P_t(\tau_{Di} \neq \tau_{Ci} \& \tau_{Ci} = t + \Delta t)$: probability that firm exits in the next period due to other reasons

The likelihood function (Cont'd)

$$P_t(\tau_{Ci} > t + \Delta t) = \exp\left[-(\lambda_{it} + \phi_{it})\Delta t\right]$$
$$P_t(\tau_{Di} = \tau_{Ci} = t + \Delta t) = 1 - \exp\left[-\lambda_{it}\Delta t\right]$$
$$P_t(\tau_{Di} \neq \tau_{Ci} \& \tau_{Ci} = t + \Delta t) = \exp[-\lambda_{it}\Delta t] - \exp\left[-(\lambda_{it} + \phi_{it})\Delta t\right]$$

with $\Delta t = 1/12$ (monthly data)

Note that the likelihood function is decomposable so that the parameters for the default and other exit intensity functions can be separately estimated.

Forward intensity model

- Spot combined exit intensity: "average" rate of combined exit occurrence

$$\psi_{it}(\tau) \equiv -\frac{\ln(1 - F_{it}(\tau))}{\tau} = -\frac{\ln E_t\left[\exp\left(-\int_t^{t+\tau}(\lambda_{is} + \phi_{is})ds\right)\right]}{\tau}$$

$F_{it}(\tau)$: the time-t conditional distribution function of the combined exit time evaluated at $t + \tau$.
λ_{is}: instantaneous intensity for default.
ϕ_{is}: instantaneous intensity for other exit.

Forward intensity model (Cont'd)

- Forward exit intensity: forward rate of combined exit occurrence

$$g_{it}(\tau) \equiv \frac{F'_{it}(\tau)}{1 - F_{it}(\tau)} = \psi_{it}(\tau) + \psi'_{it}(\tau)\tau$$

- Forward default intensity: forward rate of default occurrence

$$\begin{aligned} f_{it}(\tau) &\equiv e^{\psi_{it}(\tau)\tau} \lim_{\Delta t \to 0} \frac{P_t(t + \tau < \tau_{Di} = \tau_{Ci} \leq t + \tau + \Delta t)}{\Delta t} \\ &= e^{\psi_{it}(\tau)\tau} \lim_{\Delta t \to 0} \frac{E_t \left[\int_{t+\tau}^{t+\tau+\Delta t} \exp\left(-\int_t^s (\lambda_{iu} + \phi_{iu}) du\right) \lambda_{is} ds \right]}{\Delta t} \end{aligned}$$

τ_{Di}: default time of the i-th firm.
τ_{Ci}: combined exit time of the i-th firm.

- The default probability over $[t, t + \tau]$ becomes $\int_0^\tau e^{-\psi_{it}(s)s} f_{it}(s) ds$.

Forward intensity model (Cont'd)

- Model $f_{it}(\tau)$ and $g_{it}(\tau)$ directly as functions of state variables available at time t and the horizon of interest, τ.
- $g_{it}(\tau) \geq f_{it}(\tau) \geq 0$
- $X_{it} = (x_{it,1}, x_{it,2}, \cdots, x_{it,k})$: the set of the state variables

$$
\begin{aligned}
f_{it}(\tau) &= \exp\{\lambda(\tau)\exp[-\delta(\tau)(t-t_B)]1_{t>t_B} \\
&\quad +\alpha_0(\tau)+\alpha_1(\tau)x_{it,1}+\alpha_2(\tau)x_{it,2}+\cdots\alpha_k(\tau)x_{it,k}\} \\
g_{it}(\tau) &= f_{it}(\tau)+\exp\left(\beta_0(\tau)+\beta_1(\tau)x_{it,1}+\beta_2(\tau)x_{it,2}+\cdots\beta_k(\tau)x_{it,k}\right)
\end{aligned}
$$

  where t_B is August 2008 (Note that the US government bailed out AIG in September 2008)
- Discretize the model for empirical implementation

The pseudo-likelihood function

$$\mathscr{L}_\tau(\alpha,\beta;\tau_C,\tau_D,X)=\prod_{i=1}^{N}\prod_{t=0}^{T-1}\mathscr{L}_{\tau,i,t}(\alpha,\beta)$$

$$\begin{aligned}\mathscr{L}_{\tau,i,t}(\alpha,\beta)=&1_{\{t_{0i}\le t,\tau_{Ci}>t+\tau\}}P_t(\tau_{Ci}>t+\tau)\\&+1_{\{t_{0i}\le t,\tau_{Di}=\tau_{Ci}\le t+\tau\}}P_t(\tau_{Ci};\tau_{Di}=\tau_{Ci}\le t+\tau)\\&+1_{\{t_{0i}\le t,\tau_{Di}\neq\tau_{Ci},\tau_{Ci}\le t+\tau\}}P_t(\tau_{Ci};\tau_{Di}\neq\tau_{Ci}\&\tau_{Ci}\le t+\tau)\\&+1_{\{t_{0i}>t\}}+1_{\{\tau_{Ci}\le t\}}\end{aligned}$$

- $P_t(\tau_{Ci}>t+\tau)$: probability of surviving both forms of exit over the defined interval
- $P_t(\tau_{Ci};\tau_{Di}=\tau_{Ci}\le t+\tau)$: probability that firm defaults at a particular period within the defined interval
- $P_t(\tau_{Ci};\tau_{Di}\neq\tau_{Ci}\&\tau_{Ci}\le t+\tau)$: probability that firm exits due to other reasons at a particular period within the defined interval

The pseudo-likelihood function (Cont'd)

$$P_t(\tau_{Ci} > t+\tau) = \exp\left[-\sum_{s=0}^{\tau-1} g_{it}(s)\Delta t\right]$$

$$P_t(\tau_{Ci};\tau_{Di} = \tau_{Ci} \leq t+\tau)$$

$$= \begin{cases} 1-\exp\left[-f_{it}(0)\Delta t\right], & \text{if } \tau_{Di} = t+1 \\ \exp\left[-\sum_{s=0}^{\tau_{Di}-t-2} g_{it}(s)\Delta t\right]\left\{1-\exp\left[-f_{it}(\tau_{Di}-t-1)\Delta t\right]\right\}, & \text{if } t+1 < \tau_{Di} \leq t+\tau \end{cases}$$

$$P_t(\tau_{Ci};\tau_{Di} \neq \tau_{Ci} \& \tau_{Ci} \leq t+\tau)$$

$$= \begin{cases} \exp[-f_{it}(0)\Delta t] - \exp[-g_{it}(0)\Delta t], & \text{if } \tau_{Ci} = t+1 \\ \exp\left[-\sum_{s=0}^{\tau_{Ci}-t-2} g_{it}(s)\Delta t\right] \times & \\ \{\exp[-f_{it}(\tau_{Ci}-t-1)\Delta t] - \exp[-g_{it}(\tau_{Ci}-t-1)\Delta t]\}, & \text{if } t+1 < \tau_{Ci} \leq t+\tau \end{cases}$$

with $\Delta t = 1/12$ (monthly data)

Estimating the Forward Intensity Model

- It is an overlapped pseudo-likelihood function when the intended prediction horizon is greater than one basic time period (i.e., one month in our empirical implementation).
- The pseudo-likelihood function is decomposable so that estimation can be performed one forward period at a time.
- The pseudo-likelihood function continues to be decomposable to allow for separate estimations of the default intensity and the intensity for other form of exit.
- Because the numerical problem is non-sequential, it can be easily parallelized in computing.
- Note that the forward intensity function corresponding to $\tau = 0$ is the spot intensity function.

Data

- Sample period: 1991-2011.
- Database:
  - Compustat
  - CRSP
  - Credit Research Initiative database
- 12,268 U.S. public companies (both industrial and financial), 1,104,963 firm-month observations.

Year	Active Firms	Defaults	(%)	Other Exit	(%)
1991	4012	32	0.80%	257	6.41%
1992	4009	28	0.70%	325	8.11%
1993	4195	25	0.60%	206	4.91%
1994	4433	24	0.54%	273	6.16%
1995	5069	19	0.37%	393	7.75%
1996	5462	20	0.37%	463	8.48%
1997	5649	44	0.78%	560	9.91%
1998	5703	64	1.12%	753	13.20%
1999	5422	77	1.42%	738	13.61%
2000	5082	104	2.05%	616	12.12%
2001	4902	160	3.26%	577	11.77%
2002	4666	81	1.74%	397	8.51%
2003	4330	61	1.41%	368	8.50%
2004	4070	25	0.61%	302	7.42%
2005	3915	24	0.61%	291	7.43%
2006	3848	15	0.39%	279	7.25%
2007	3767	19	0.50%	352	9.34%
2008	3676	59	1.61%	285	7.75%
2009	3586	67	1.87%	244	6.80%
2010	3396	25	0.74%	242	7.13%
2011	3224	21	0.65%	226	7.01%

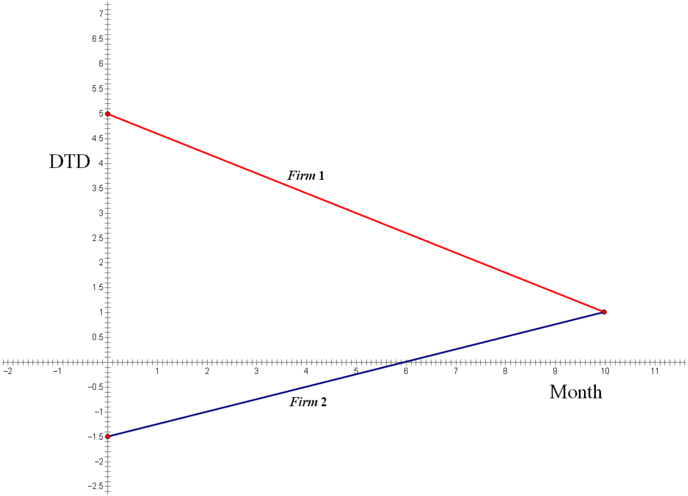
Covariates

- An exponential decaying term to capture the US intervention effect
- 3-month treasury rate
- Trailing 1-year S&P500 return
- Distance to default
- Cash and short-term investments/Total assets
- Net income/Total assets
- Relative size
- Market to book ratio
- Idiosyncratic volatility

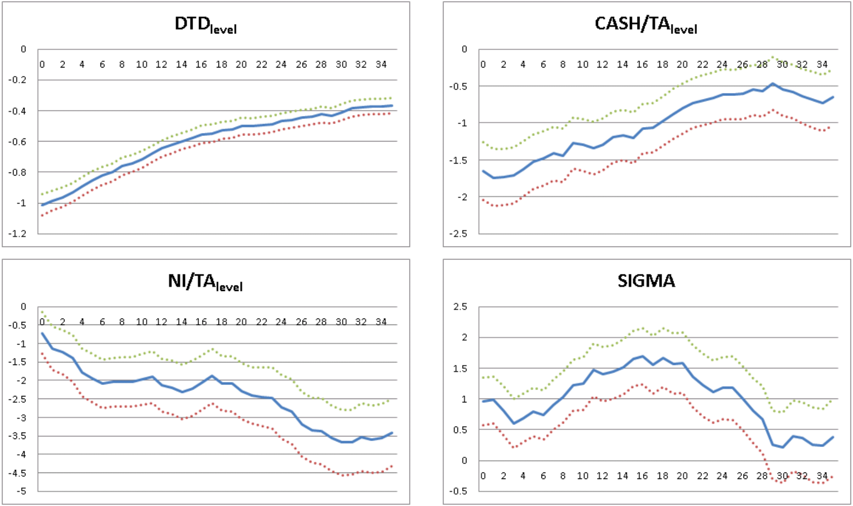
Note: Refer to Duan and Wang (2012) for estimating DTDs for non-financial and financial firms.

Covariates (Cont'd)

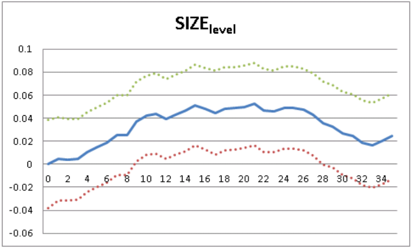
- Level and trend

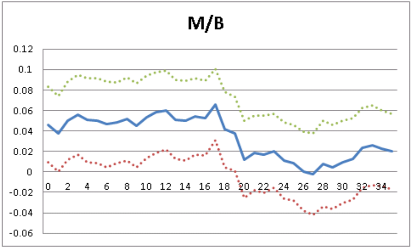


Parameter Estimates

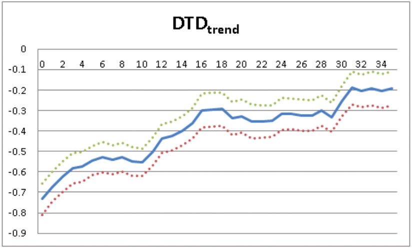


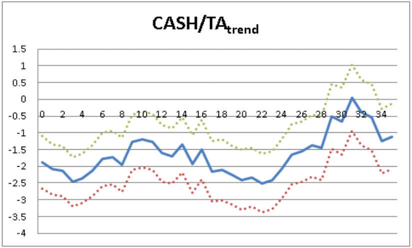
Parameter Estimates (Cont'd)

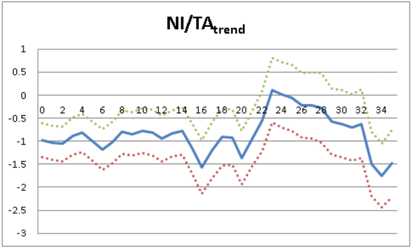


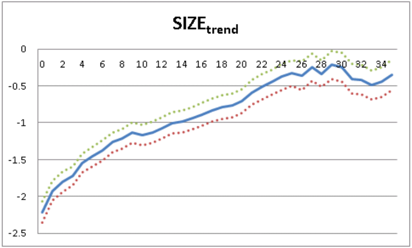


Parameter Estimates (Cont'd)

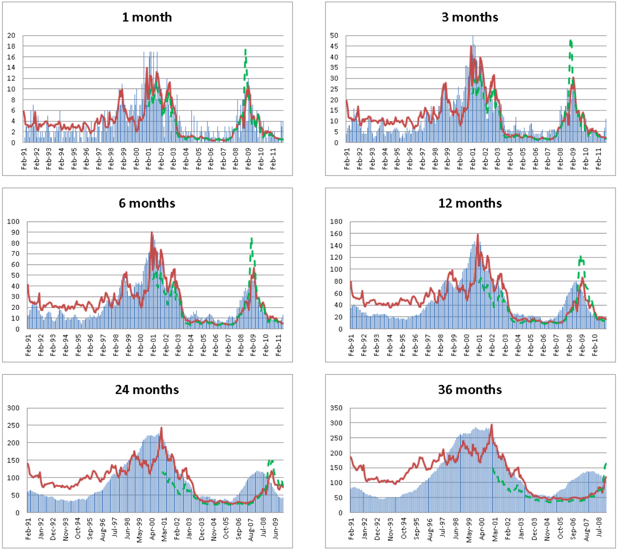




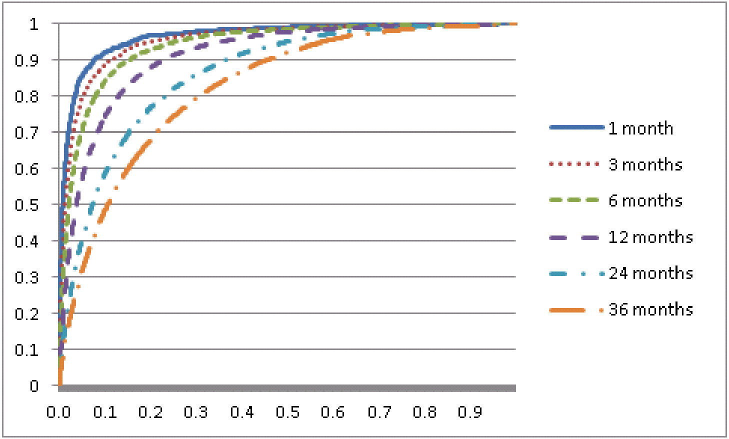




Aggregate Number of Defaults

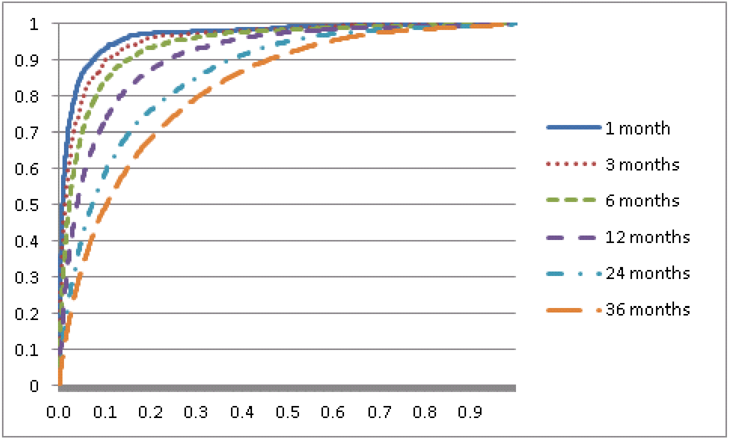


In-Sample Accuracy



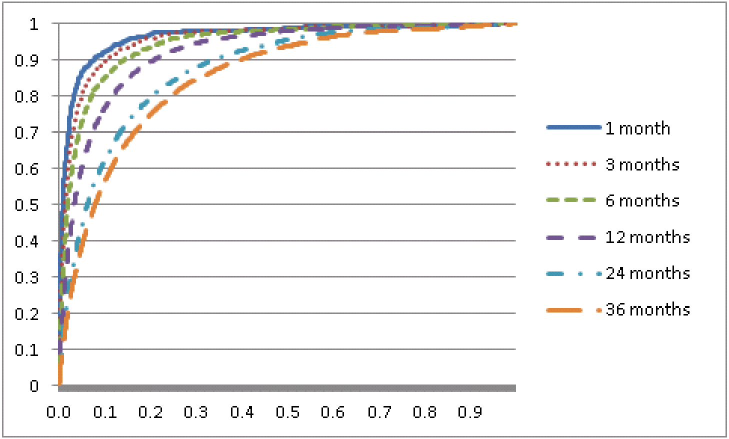
1 month	3 months	6 months	12 months	24 months	36 months
93.22%	91.30%	88.63%	83.52%	74.10%	66.67%

Out-of-Sample (Cross-Section) Accuracy



1 month	3 months	6 months	12 months	24 months	36 months
93.77%	91.74%	88.88%	83.36%	73.37%	65.47%

Out-of-Sample (Over Time) Accuracy

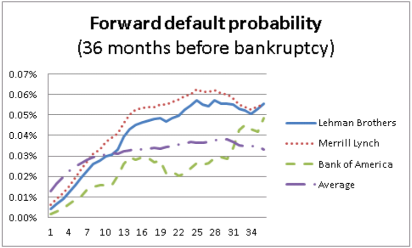


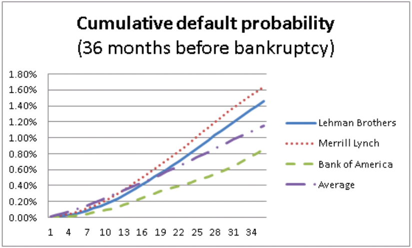
1 month	3 months	6 months	12 months	24 months	36 months
93.31%	91.81%	89.42%	85.16%	76.43%	72.45%

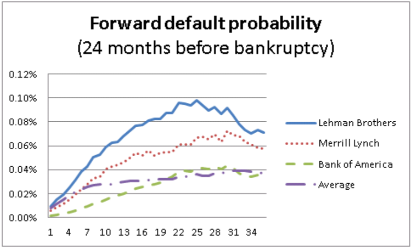
Accuracy Ratios with/without Smoothing

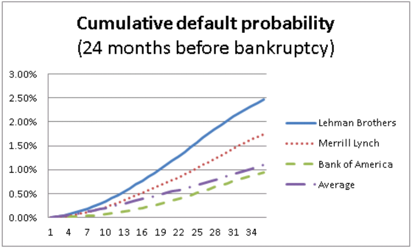
Panel A: In-sample result						
	1 month	3 months	6 months	12 months	24 months	36 months
Full sample	93.22%	91.30%	88.63%	83.52%	74.10%	66.67%
Full sample (smoothed)	93.29%	91.35%	88.65%	83.51%	74.07%	66.66%
Non-financial	93.21%	91.18%	88.32%	82.99%	73.96%	66.98%
Financial	93.03%	91.59%	90.57%	87.38%	74.18%	59.88%
Panel B: Out-of-sample (cross-section) result						
	1 month	3 months	6 months	12 months	24 months	36 months
Full sample	93.77%	91.74%	88.88%	83.36%	73.37%	65.47%
Full sample (smoothed)	93.61%	91.69%	88.88%	83.39%	73.33%	65.43%
Non-financial	93.69%	91.58%	88.50%	82.76%	73.16%	65.84%
Financial	91.28%	89.42%	88.70%	85.64%	71.65%	55.57%
Panel C: Out-of-sample (over time) result						
	1 month	3 months	6 months	12 months	24 months	36 months
Full sample	93.31%	91.81%	89.42%	85.16%	76.43%	72.45%
Full sample (smoothed)	93.51%	91.91%	89.37%	85.02%	76.52%	72.42%
Non-financial	93.73%	92.27%	89.80%	85.37%	77.11%	73.03%
Financial	92.70%	91.65%	91.06%	88.62%	78.04%	73.38%

Forward and Cumulative Term Structures of PDs

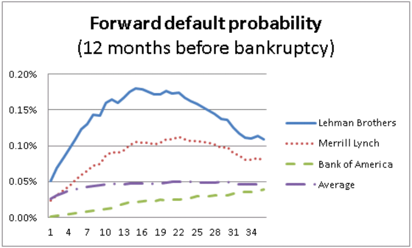


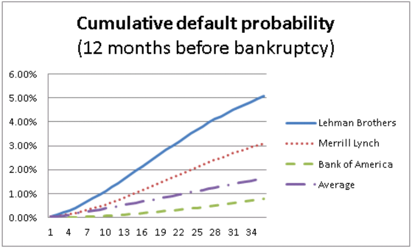


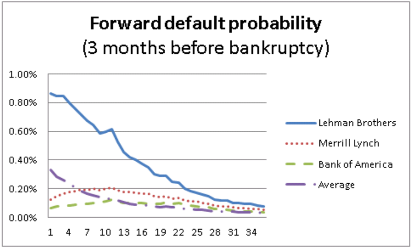


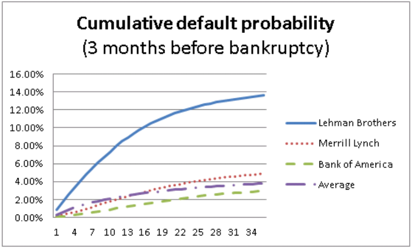


Forward and Cumulative Term Structures of PDs (Cont'd)









Duan, *et al* (2012) vs. Duffie, *et al* (2007)

Apply 4 covariates (trailing 1-year S&P 500 index return, 3-month treasury rate, firms' distance-to-default and firms' 1-year stock return)

Panel A: In-sample result (1991-2011)						
	1 month	3 months	6 months	12 months	24 months	36 months
Duffie, *et al* (2007)	91.95%	90.06%	88.14%	85.37%	80.54%	77.22%
Duffie, *et al* (2007) (restricted)	91.95%	89.96%	87.24%	81.72%	71.28%	63.85%
Duan, *et al* (2012)	91.95%	89.63%	86.78%	81.43%	71.43%	64.01%
Panel B: In-sample result (2001-2011)						
	1 month	3 months	6 months	12 months	24 months	36 months
Duffie, *et al* (2007)	92.26%	91.08%	89.19%	86.58%	81.22%	77.58%
Duffie, *et al* (2007) (restricted)	92.26%	91.12%	88.91%	84.58%	75.04%	68.98%
Duan, *et al* (2012)	92.26%	90.85%	88.56%	84.68%	76.15%	70.39%
Panel C: Out-of-sample (over time) result (2001-2011)						
	1 month	3 months	6 months	12 months	24 months	36 months
Duffie, *et al* (2007)	91.97%	91.38%	87.43%	77.50%	60.33%	51.87%
Duffie, *et al* (2007) (restricted)	91.97%	90.80%	88.44%	83.52%	71.66%	65.04%
Duan, *et al* (2012)	91.97%	90.50%	88.04%	83.77%	74.67%	70.31%

Conclusion

- A forward intensity approach works better for the prediction of corporate defaults over different future periods.
- Several frequently used covariates are shown to be useful for prediction at both short and long horizons.
- The results confirm the bailout effect and the forward intensity models captures the Lehman Brothers episode remarkably well.
- The forward intensity model is amenable to aggregation, which allows analysts to assess default behavior at the portfolio and/or economy level.

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