Ext-ANALOGUE OF BRANCHING LAWS FOR CLASSICAL GROUPS

Considering the restriction of representations, say of $SO(n+1)$ to $SO(n)$ over a non-Archimedean local field has been a very fruitful direction of research about which many results have recently been proven. The question amounts to understanding $\mathrm{Hom}_{SO(n)}[\pi_1, \pi_2]$ for irreducible admissible representation π_1 of $SO(n+1)$, and π_2 of $SO(n)$. Given the interest in the space $\mathrm{Hom}_{SO(n)}[\pi_1, \pi_2]$, it is natural to consider the other related spaces $\mathrm{Ext}^i_{SO(n)}[\pi_1, \pi_2]$, and in fact homological algebra methods suggest that the objects for which one might expect simplest answers are not these individual groups, but the alternating sum of their dimensions:

$$EP(\pi_1, \pi_2) = \sum_{i=0}^{i=d} (-1)^i \dim \mathrm{Ext}^i_{SO(n)}[\pi_1, \pi_2],$$

where d is the rank of $SO(n)$.

We discuss some of this in the lecture.