Inverse Polynomial Optimization

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Let $\mathbf{P}:\ \min\{f(\mathbf{x}) : \mathbf{x}\in\mathbf{K}\}$ be a polynomial optimization problem where $\mathbf{K}\subset\mathbb{R}^n$ is a compact basic semi-algebraic set and $f\in\mathbb{R}[\mathbf{x}]$. The *inverse optimisation* problem associated with $\mathbf{P}$ and a given feasible point $\mathbf{y}\in\mathbf{K}$, consists of solving :

$$\mathrm{INV}{-}\mathbf{P}:\quad \min_{\tilde{f}\in\mathbb{R}[\mathbf{x}]}\{\,\|\tilde{f}-f\|\,:\,\mathbf{y}=\arg\min_{\mathbf{x}}\{\tilde{f}(\mathbf{x})\,:\,\mathbf{x}\in\mathbf{K}\,\}\,\},$$

where $\|\cdot\|$ is a norm of $\mathbb{R}[\mathbf{x}]$. That is, one searches for a polynomial $\tilde{f}\in\mathbb{R}[\mathbf{x}]$, as close to f as possible, and for which the given point $\mathbf{y}\in\mathbf{K}$ is a global optimal solution of $\min\{\tilde{f}(\mathbf{x}) : \mathbf{x}\in\mathbf{K}\}$.

• We first provide a numerical scheme for solving INV$-\mathbf{P}$. In particular, one computes a polynomial $\tilde{f}_d$ with a-priori fixed degree d ($=\deg f$ if desired) and with the following properties :
(a) $\mathbf{y}\in\mathbf{K}$ is a global optimal solution of $\min\{\tilde{f}\,:\,\mathbf{x}\in\mathbf{K}\}$, with a Putinar's certificate of optimality of size bounded a-priori.
(b) $\tilde{f}_d$ minimizes $\|g-f\|$ among all $g\in\mathbb{R}[\mathbf{x}]_d$ having property (a) :

• In addition, $\tilde{f}_d$ is an optimal solution of an SDP whose size is controlled by the a-priori maximum size imposed to the Putinar's certificate of optimality. Finally, we show that when minimizing the ℓ_1-norm $\|g-f\|_1$, one obtains a *canonical sparse* optimal solution $\tilde{f}_d$ whose explicit form is provided.