Efficient Computation of Weights for Lattice Rules in High Dimensional Integration

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Abstract.

Randomly shifted lattice rules are one of main techniques of Quasi-Monte Carlo for high dimensional integration. They are an efficient integration method in certain examples of two-stage stochastic programs, where the rate of convergence can be $O(\frac{1}{N})$ instead of the classical Monte Carlo convergence rate $O(\frac{1}{\sqrt{N}})$, even when the integrands are functions presenting "kinks".

Modern lattices are constructed based on a set of parameters called "weights". The weights define the embedding of a function on particular weighted (unanchored) Sobolev space, and are used by the component–by–component construction of generator vectors for lattice rules. If the function at hand exhibits low effective dimension in truncation or superposition sense, (mixed-)partial derivative information can be used to approximate the (semi)norm of the ANOVA effective part of the function, resulting in a simplification to fix good weights. We use new methods for accurate evaluation and bounding of mixed derivatives of functions $f : \mathbb{R}^d \to \mathbb{R}$ by means of algorithmic differentiation (AD). We show numerical results comparing different type of weights in examples related to finance.