Reduction of linear complementarily problems to bimatrix games

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In 1994, Christos Papadimitriou introduced the complexity class *PPAD* (*Polynomial-time Parity Argument Directed)* for problems whose solution is known to exist via a proof based on a certain directed acyclic graph. In particular, we will show in this talk that the majority of linear complementarily problems (LCP's) which are processable by the Lemke algorithm can be shown to be in *PPAD*. The discovery that finding a Nash equilibrium for a bimatrix game (2-NASH) is *PPAD*-complete established the very surprising result that every LCP in *PPAD* can be reduced in polynomial time to a 2-NASH (which can, by itself, be formulated as an LCP). However, the ingeniously constructed reduction (which is designed for any *PPAD* problem) is very complicated and goes through several stages that involve reducing the given LCP to finding an approximate Brouwer fixed point of an appropriate function, followed by reducing the latter to 3-graphical NASH (using small polymatrix games to simulate the computation of certain simple arithmetic operations), and finally, reducing the 3-graphical NASH to 2-NASH. Thus, while of great theoretical significance, the reduction is not practical for actually solving an LCP via 2-NASH, and it does not provide a clear insight regarding the completeness of 2-NASH within the *PPAD* LCP's. To address this concern, we develop a simple explicit reduction of Lemke processable LCP's to 2-NASH problems. In particular,we show that the reduction is a bijection and discuss its implications for solving LCP's via 2-NASH and for getting a deeper insight into these LCP's.