Formulation, degeneracy, and finite precision: lemke's method for strictly positive matrices

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Abstract.

In this talk, we consider computing a nonzero solution to the linear complementarity problem $0 \leq x \perp Mx + q$ where $q > 0$. We present several formulations of this problem and their properties including one reformulation with a strictly positive matrix for which Lemke's method with appropriate degeneracy resolution is guaranteed to compute a solution. We then apply the standard Lemke method with a ray start as implemented in PATH to random instances from this problem class and detect physical cycles of nonzero length in the path constructed. Further analysis shows that the cycling is due to the DEVEX and EXPAND pivot selection rules, which slightly perturb the path allowing one to jump off the true path. This observation leads to the conjecture that Lemke's method with a standard lexicographic pivot selection rule for degeneracy resolution can cycle in finite precision arithmetic, due to similar perturbations; only in infinite precision arithmetic is the assertion that the path contains no cycles true. For the random instances from the problem class considered, however, Lemke's method with a random covering vector and the textbook pivot selection rule can efficiently compute solutions.