Optimality Conditions and Finite Convergence of Lasserre's Hierarchy

Jiawang Nie

Workshop on Large Scale Conic Optimization
Institute for Mathematical Sciences, NUS, Singapore

Mathematics Department, UCSD, USA

November 21, 2012

Multivariate Polynomial Optimization

Given polynomials $f(x), h_i(x), g_j(x)$, we want to solve

$$\begin{array}{rl} \min\limits_{x\in\mathbb{R}^n} & f(x) \\ s.t. & h_1(x) = \cdots = h_{m_1}(x) = 0, \\ & g_1(x) \geq 0, \cdots, g_{m_2}(x) \geq 0. \end{array}$$

Lasserre's hierarchy is a sequence of sum of squares (SOS) relaxations for solving the problem globally. He proved asymptotic convergence under a condition on (h, g).

Question: How often does finite convergence occur?

Answer: Almost always! (The goal of this talk.)

Outline of the Talk

- Introduction of Lasserre's Hierarchy
- Optimality Conditions and Finite Convergence
- Genericity of Optimality Conditions
- Certifying Finite Convergence

SOS polynomials

A polynomial p is sum of squares (SOS) if $p = \sum q_i^2(x)$.

$$\text{Example:} \quad 3 \cdot \left(x_1^4 + x_2^4 + x_3^4 + x_4^4 - 4x_1x_2x_3x_4\right)$$
$$= (x_1^2 - x_2^2 - x_4^2 + x_3^2)^2 + (x_1^2 + x_2^2 - x_4^2 - x_3^2)^2 +$$
$$(x_1^2 - x_2^2 - x_3^2 + x_4^2)^2 + 2(x_1x_4 - x_2x_3)^2 +$$
$$2(x_1x_2 - x_3x_4)^2 + 2(x_1x_3 - x_2x_4)^2$$

SOS implies nonnegativity, but not conversely.

Theorem (Hilbert, 1888) Every nonnegative poly is SOS iff

$$(\# \, var, degree) = (1, 2d), (*, 2), \text{ or } (2, 4).$$

Hilbert'1 17th Problem: Is every nonnegative poly is a sum of squares of rational functions? (Yes, by Artin).

Testing SOS Membership

A polynomial p is SOS if and only if (Lasserre, Parrilo, ...)

$$\exists\, X: \qquad p = [x]_d^T X [x]_d, \quad X = X^T \succeq 0.$$

The X is called a Gram matrix.

$$\begin{aligned} & 2x_1^4 + 2x_1^3 x_2 - x_1^2 x_2^2 + 5x_2^4 \\ = & \begin{bmatrix} x_1^2 \\ x_2^2 \\ x_1 x_2 \end{bmatrix}^T \begin{bmatrix} 2 & 0 & 1 \\ 0 & 5 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1^2 \\ x_2^2 \\ x_1 x_2 \end{bmatrix} \\ = & \begin{bmatrix} x_1^2 \\ x_2^2 \\ x_1 x_2 \end{bmatrix}^T \begin{bmatrix} 2 & -\alpha & 1 \\ -\alpha & 5 & 0 \\ 1 & 0 & -1 + 2\alpha \end{bmatrix} \begin{bmatrix} x_1^2 \\ x_2^2 \\ x_1 x_2 \end{bmatrix} \end{aligned}$$

When $\alpha = 3$, the Gram matrix is positive semidefinite.

Truncated ideals and qudratic modules.

Let $h = (h_1, \dots, h_{m_1})$ and $g = (g_1, \dots, g_{m_2})$.

The $2k$-th truncated ideal generated by h is

$$\langle h \rangle_{2k} := \left\{ \sum_{i=1}^{m_1} \phi_i h_i \,\middle|\, \begin{array}{c} \text{each } \phi_i \in \mathbb{R}[x] \\ \text{and } \deg(\phi_i h_i) \le 2k \end{array} \right\}.$$

The k-th truncated quadratic module generated by g is ($g_0 = 1$)

$$Q_k(g) := \left\{ \sum_{j=0}^{m_2} \sigma_j g_j \,\middle|\, \begin{array}{c} \text{each } \sigma_j \in \mathbb{R}[x] \text{ is SOS} \\ \text{and } \deg(\sigma_j g_j) \le 2k \end{array} \right\}.$$

The set $\langle h \rangle_{2k}$ is a subspace and $Q_k(g)$ is an SDP set.

Lasserre's Hierarchy

$$\begin{array}{ll} \min & f(x) \\ s.t. & (h_1, \ldots, h_{m_1})|_x = 0,\ (g_1, \ldots, g_{m_2})|_x \geq 0. \end{array}$$

For a relaxation order k, solve the SOS program (by SDP)

$$\begin{array}{lll} f_k := \max & \gamma \\ & s.t. & f - \gamma = \phi + \sigma,\ \phi \in \langle h \rangle_{2k}, \sigma \in Q_k(g). \end{array}$$

For $k = 1, 2, \ldots$, we get a sequence of lower bounds:

$$f_1 \leq f_2 \leq f_3 \leq \cdots \leq f_{min}.$$

Lasserre's hierarchy is the sequence $\{f_k\}$ of lower bounds.

Lasserre's hierarchy has finite convergence if

$$f_k = f_{min} \qquad \text{for all } k \text{ big enought.}$$

Lasserre's Relax.: Example 1

$$\min_{x_1,x_2} \quad -(x_1-1)^2-(x_1-x_2)^2-(x_2-3)^2$$
$$s.t. \quad 1-(x_1-1)^2 \geq 0, 1-(x_1-x_2)^2 \geq 0, 1-(x_2-3)^2 \geq 0$$

Applying Lasserre's relax., we get

order k	lower bound f_k	minimum f_{min}	minimizer
1	-3	-2	not found
2	-2	-2	(1,2)

Lasserre's hierarchy converges at $k = 2$.

Lasserre's Relax.: Example 2

$$
\begin{aligned}
\min_{x_1,x_2} \quad & -x_1 - x_2 \\
s.t. \quad & x_2 \le 2x_1^4 - 8x_1^3 + 8x_1^2 + 2 \\
& x_2 \le 4x_1^4 - 32x_1^3 + 88x_1^2 - 96x_1 + 36 \\
& 0 \le x_1 \le 3,\ 0 \le x_2 \le 4
\end{aligned}
$$

Applying Lasserre's relax., we get

order k	lower bound f_k	minimum f_{min}	minimizer
2	-7	-5.5080	not found
3	-6.667	-5.5080	not found
4	-5.5080	-5.5080	(2.3295,3.1785)

Lasserre's hierarchy converges at $k = 4$.

Convergence of Lasserre's Hierarchy

Poly Optim. Prob.

$$\begin{array}{rl} f_{min} := & \min f(x) \\ s.t. & (h_1, \ldots, h_{m_1})|_x = 0 \\ & (g_1, \ldots, g_{m_2})|_x \geq 0 \end{array}$$

k-th Lasserre's relax.

$$\begin{array}{rl} f_k := & \max \gamma \\ s.t. & f - \gamma = \phi + \sigma, \\ & \phi \in \langle h \rangle_{2k}, \sigma \in Q_k(g). \end{array}$$

Archimedean condition (AC): $\exists\, R > 0, t \in \mathbb{N}$ such that

$$R - \|x\|_2^2 \in \langle h \rangle_{2t} + Q_t(g).$$

AC is almost equivalent to compactness of the feasible set.

Theorem (Lasserre, 2001) If AC holds, then

$$\lim_{k \to \infty} f_k = f_{min} \quad \text{(asymptotic convergence).}$$

Known Results about Finite Convergence

Lassrre hierarchy has finite convergence (i.e., $f_k = f_{min}$ for all k big enough) in the following cases:

• $V_{\mathbb{C}}(h)$ is finite (Laurent, 2007). If only $V_{\mathbb{R}}(h)$ is finite, the dual (moment) version has finite convergence (Laurent, 2009).

• $V_{\mathbb{R}}(h)$ is a finite set (N., 2012).

• The objective and constraints are strictly convex on the feasible set (Lasserre, 2009)

• The objective and constraints are convex on the feasible set and strictly convex at global minimizers (Laurent-de Klerk, 2011)

• Generic cases (this talk)

Finite versus Asymptotic Convergence

Bad news (Scheiderer): If the feasible set has dimension ≥ 3, there always exists a "bad" f such that

$$\lim_{k\to\infty} f_k = f_{min}, \qquad \text{but} \qquad f_k < f_{min} \quad \forall\, k.$$

Good news: If f is not specially chosen (e.g., randomly generated), we *almost always* have (numerically demonstrated)

$$f_k = f_{min} \qquad \text{if } k \text{ is big enough.}$$

Question: Why does Lasserre's hierarchy work so well in practice?

Goal of this talk: Give an interpretation for the performance!

Outline of the Talk

- Introduction of Lasserre's Hierarchy
- Optimality Conditions and Finite Convergence
- Genericity of Optimality Conditions
- Certifying Finite Convergence

First order optimality condition (FOOC)

Let u be a local minimizer of

$$\begin{array}{rl} \min & f(x) \\ s.t. & (h_1, \ldots, h_{m_1})|_x = 0,\ (g_1, \ldots, g_{m_2})|_x \geq 0. \end{array}$$

Let $J(u) = \{j_1, \ldots, j_r\}$ be the active set at u. If the *constraint qualification condition (CQC)* holds at u, i.e.,

$$\nabla h_1(u), \ldots, \nabla h_{m_1}(u), \nabla g_{m_1}(u), \ldots, \nabla g_{j_r}(u)$$

are linearly independent, then there exist Lagrange multipliers $\lambda_1, \ldots, \lambda_{m_1}$ and $\mu_1, \ldots, \mu_{m_2}$ satisfying

$$\nabla f(u) = \sum_{i=1}^{m_1} \lambda_i \nabla h_i(u) + \sum_{j=1}^{m_2} \mu_j \nabla g_j(u).$$

Complementarity Conditions

Let u be a local minimizer of

$$\begin{array}{ll} \min & f(x) \\ s.t. & (h_1, \ldots, h_{m_1})|_x = 0,\ (g_1, \ldots, g_{m_2})|_x \geq 0. \end{array}$$

Let $J(u) = \{j_1, \ldots, j_r\}$ be the active set at u.

The Lagrange multipliers $\lambda_1, \ldots, \lambda_{m_1}$ and $\mu_1, \ldots, \mu_{m_2}$ in FOOC further satisfies the complementarity condition

$$\mu_1 g_1(u) = \cdots = \mu_{m_2} g_{m_2}(u) = 0, \quad \mu_1 \geq 0, \ldots, \mu_{m_2} \geq 0.$$

The strict complementarity condition (SCC) holds at u if

$$\mu_1 + g_1(u) > 0, \ldots, \mu_{m_2} + g_{m_2}(u) > 0$$

which is equivalent to

$$\mu_j > 0 \qquad \forall\, j \in J(u).$$

Second Order Necessity Condition (SONC)

Let $L(x)$ be the Lagrange function

$$L(x) := f(x) - \sum_{i=1}^{m_1} \lambda_i h_i(x) - \sum_{j=1}^{m_2} \mu_j \mu_j g_j(x)$$

where λ_i and μ_j are Lagrange multipliers.

SONC at a local minimizer u states that

$$v^T \nabla_x^2 L(u) v \geq 0 \qquad \text{for all } v \in G(u)^{\perp},$$

$$G(x) := \begin{bmatrix} \nabla h_1(x) & \cdots & \nabla h_{m_1}(x) & \nabla g_{m_1}(x) & \cdots & \nabla g_{j_r}(x) \end{bmatrix}^T.$$

$\nabla_x^2 L(u)$ is positive semidefinite in the tangent space $G(u)^{\perp}$.

Second Order Sufficiency Condition (SOSC)

Let $L(x)$ be the Lagrange function

$$L(x) := f(x) - \sum_{i=1}^{m_1} \lambda_i h_i(x) - \sum_{j \in J(u)} \mu_j g_j(x)$$

where λ_i and μ_j are Lagrange multipliers.

SOSC at a local minimizer u requires that

$$v^T \nabla_x^2 L(u) v > 0 \qquad \text{for all } 0 \neq v \in G(u)^{\perp}.$$

$\nabla_x^2 L(u)$ is positive definite in the tangent space $G(u)^{\perp}$.

Theory of Optimality Conditions in NLP

Necessity:

$$\text{CQC} + \text{local optimality} \Rightarrow \text{FOOC} + \text{CC} + \text{SONC}.$$

Sufficiency:

$$\text{FOOC} + \text{SCC} + \text{SOSC} \Rightarrow \text{strict local optimality}.$$

Optimality Conditions implies Finite Convergence

Theorem (N.,2012): Under archimedean condition (AC), if CQC, SCC and SOSC hold at every global minimizer of f on K, then

$$f_k = f_{min} \quad \text{for all } k \text{ big enough.}$$

(Lasserre's hierarchy has finite convergence.)

Counterexamples exist if one of AC, CQC, SCC, SOSC fails.

$$\begin{array}{ll} \min & x_1^6 + x_2^6 + x_3^6 + 3x_1^2x_2^2x_3^2 - x_1^4(x_2^2 + x_3^2) - x_2^4(x_3^2 + x_1^2) - x_3^4(x_1^2 + x_2^2) \\ s.t. & x_1^2 + x_2^2 + x_3^2 = 1. \end{array}$$

The global minimizers are

$$\frac{1}{\sqrt{3}}(\pm 1, \pm 1, \pm 1), \frac{1}{\sqrt{2}}(\pm 1, \pm 1, 0), \frac{1}{\sqrt{2}}(\pm 1, 0, \pm 1), \frac{1}{\sqrt{2}}(0, \pm 1, \pm 1).$$

CQC, SCC, SOSC are satisfied, e.g., at $u = \frac{1}{\sqrt{3}}(1, 1, 1)$,

$$\nabla_x^2 L(u) = \frac{4}{9}\left(3 \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}^T \right), \quad G(u)^\perp = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}^\perp.$$

The Lasserre's hierarchy has finite convergence:

$$f_k = f_{min} = 0 \qquad \forall\, k \geq 5 \qquad \text{(by GloptiPoly 3).}$$

Sketchy Proof of the main Theorem

Theorem (N.,2012): Under archimedean condition (AC), if CQC, SCC and SOSC hold at every global minimizer of f on K, then

$$f_k = f_{min} \quad \text{for all } k \text{ big enough.}$$

(i.e., Lasserre's hierarchy has finite convergence.)

Proof. The CQC, SCC,SOSC imply that the boundary Hessian condition (BHC) holds at every global minimizer of f on K, and then apply the BHC theorem of Murray Marshall. □

Local Parameterization Condition (LPC)

Let $V := V_{\mathbb{R}}(h_1, \ldots, h_{m_1})$ be a real algebraic variety and

$$K = \{x \in V : g_1(x) \geq 0, \ldots, g_{m_2}(x) \geq 0\}.$$

(**Marshall,2006**) Local Parameterization Condition at u:
i) The point $u \in V$ is nonsingular;
ii) $\exists$ a neighborhood $\mathcal{O}$ of u such that $V \cap \mathcal{O}$ is parameterized by free parameters $t_1, \ldots, t_\ell$;
iii) $\exists\ 1 \leq \nu_1 < \cdots < \nu_r \leq m_2$ such that

$$t_1 = g_{\nu_1}, \ldots, t_r = g_{\nu_r} \text{ on } V \cap \mathcal{O};$$

iv) $K \cap \mathcal{O}$ is defined by $t_1 \geq 0, \ldots, t_r \geq 0$ in $(t_1, \ldots, t_r)$-coordinates.

In brief, LPC requires that a subset of $\{g_1, \ldots, g_{m_2}\}$, plus other parameters, can parameterize K locally at u.

Boundary Hessian Condition (BHC)

Let $V := V_{\mathbb{R}}(h_1, \ldots, h_{m_1})$ be a real algebraic variety and

$$K = \{x \in V : g_1(x) \geq 0, \ldots, g_{m_2}(x) \geq 0\}.$$

(**Marshall**) Boundary Hessian Condition (BHC) at u:
i) LPC holds at u; let $(t_1, \ldots, t_\ell)$ be local parameters for K s.t.

$$t_1 = g_{\nu_1}, \ldots, t_r = g_{\nu_r}, \ r \leq \ell.$$

Expand f locally around u as $f = f_0 + f_1 + f_2 + \cdots$ where each f_i is homogeneous of degree i in $(t_1, \ldots, t_\ell)$;
ii) The linear form $f_1 = a_1 t_1 + \cdots + a_r t_r$ and has positive coeffs.

$$a_1 > 0, \ldots, a_r > 0;$$

iii) Te quadratic form $f_2(0, \ldots, 0, t_{r+1}, \ldots, t_\ell)$ is positive definite in $(t_{r+1}, \ldots, t_\ell)$.

BHC Theorem

Let $V := V_{\mathbb{R}}(h_1, \ldots, h_{m_1})$ be a real algebraic variety and

$$K = \{x \in V : g_1(x) \geq 0, \ldots, g_{m_2}(x) \geq 0\}.$$

Theorem (Marshall,2006) Let f be a polynomial. Assume:
i) The archimedean condition holds for (h, g);
ii) The BHC holds at every global minimizer of f on K.
Then, $\exists\, \sigma \in Q(g)$, such that

$$f - f_{min} \equiv \sigma \quad mod \quad I(V) := \{p \in \mathbb{R}[x] : p|_V \equiv 0\}.$$

Optimality conditions $\Rightarrow$ BHC

Let $V := V_{\mathbb{R}}(h_1, \ldots, h_{m_1})$ be a real algebraic variety and

$$K = \{x \in V : g_1(x) \geq 0, \ldots, g_{m_2}(x) \geq 0\}.$$

Theorem (N.,2012) Let u be a global minimizer of f on K.
i) CQC $\Rightarrow$ LPC holds at u.
Expand f locally as $f = f_0 + f_1 + f_2 + \cdots$ in $(t_1, \ldots, t_\ell)$.
ii) FOOC and SCC imply that $f_1 = a_1 t_1 + \cdots + a_r t_r$ and

$$a_1 > 0, \ldots, a_r > 0;$$

iii) SOSC implies that $f_2(0, \ldots, 0, t_{r+1}, \ldots, t_\ell)$ is positive definite in $(t_{r+1}, \ldots, t_\ell)$.

In brief, CQC+SCC+SOSC $\Rightarrow$ BHC.

Outline of the Talk

- Introduction of Lasserre's Hierarchy
- Optimality Conditions and Finite Convergence
- Genericity of Optimality Conditions
- Certifying Finite Convergence

Genericity of Optimality conditions.

Theorem (N.,2012) There exist polynomials

$$\varphi_1, \cdots, \varphi_L$$

which are in the coefficients of f, h_i, g_j, such that if

$$\varphi_1(f, h_i, g_j) \neq 0, \ldots, \varphi_L(f, h_i, g_j) \neq 0,$$

then CQC, SCC, and SOSC hold at every local minimizer.

In brief, the optimality conditions hold in a Zariski open set.

Constraint Qualification Condition is generic

Let $K = \{x \in \mathbb{R}^n : (h_1, \ldots, h_{m_1})|_x = 0, (g_1, \ldots, g_{m_2})|_x \geq 0\}$.

Let $\{j_1, \ldots, j_r\}$ be the active set at u. If

$$\Delta(h_1, \ldots, h_{m_1}, g_{j_1}, \ldots, g_{j_r}) \neq 0,$$

($\Delta(\cdots)$ is a polynomial in the coef. of h_i, g_j), then the constraint qualification condition holds at u. This is because

$$h_1 = \cdots = h_{m_1} = g_{j_1} = \cdots = g_{j_r} = 0$$

has no singular solutions if $\Delta(h_1, \ldots, g_{j_1}, \ldots) \neq 0$.

Strict Complementarity Condition is generic

Consider the optimization problem

$$\min \quad f(x) \quad s.t. \quad h(x) = 0,\ g(x) \geq 0.$$

The KKT condition at a local minimizer u (with $g(u) = 0$) is

$$\nabla f(u) = \lambda \nabla h(u) + \mu \nabla g(u),\ h(u) = 0,\ \mu \geq 0.$$

SCC requires $\mu > 0$. If $\mu = 0$, then u is a solution of

$$\nabla f(x) = \lambda \nabla h(x),\ h(x) = 0,\ g(x) = 0.$$

If f, h has generic coef., then the polynomial system

$$\nabla f(x) = \lambda \nabla h(x),\ h(x) = 0$$

has finitely many solutions, which do not intersect $g(x) = 0$ if g is generic.

Second Order Sufficiency Condition is generic

$$\min_{x \in \mathbb{R}^n} \quad f(x).$$

The first order optimality condition is

$$f_{x_1}(x) = \cdots = f_{x_n}(x) = 0.$$

If u is a local minimizer, SOSC $\Leftrightarrow \nabla^2 f(u) \succ 0$.
($\nabla^2 f$ is the Jacobian of ∇f.)

By definition of discriminants, if

$$\Delta(f_{x_1}, \ldots, f_{x_n}) \neq 0,$$

then $\nabla f(x) = 0$ has no solution u with $\nabla^2 f(u)$ singular:

$$\Delta(f_{x_1}, \ldots, f_{x_n}) \neq 0 \;\Rightarrow\; SOSC.$$

Outline of the Talk

- Introduction of Lasserre's Hierarchy
- Optimality Conditions and Finite Convergence
- Genericity of Optimality Conditions
- Certifying Finite Convergence

How to Certify Exactness?

Lasserre's hierarchy is a sequence of lower bounds

$$f_1 \le \cdots \le f_k \le \cdots \le f_{min}$$

for the polynomial optimization

$$\begin{array}{rl} \min & f(x) \\ s.t. & (h_1, \ldots, h_{m_1})|_x = 0,\ (g_1, \ldots, g_{m_2})|_x \ge 0. \end{array}$$

The equality $f_k = f_{min}$ is certified if a feasible x^* satisfies

$$f(x^*) \quad = \quad f_k.$$

Can we always find such x^* if finite convergence occurs?

This is almost always possible!

Duality in Lasserre's Relaxations

For the polynomial optimization problem

$$\begin{array}{ll} \min & f(x) \\ s.t. & (h_1, \ldots, h_{m_1})|_x = 0, \, (g_1, \ldots, g_{m_2})|_x \geq 0. \end{array}$$

the k-th Lasserre's relaxation (SOS version) is:

$$\max \quad \gamma \quad s.t. \quad f - \gamma = \phi + \sigma g, \, \phi \in \langle h \rangle_{2k}, \, \sigma \in Q_k(g).$$

Its dual problem (moment version) is:

$$\begin{array}{lll} \min & f^T y & s.t. \; L_h^{(k)}(y) = 0 \, (1 \leq i \leq m_1), \, y_0 = 1, \\ & & \quad L_{g_j}^{(k)}(y) \succeq 0 \, (0 \leq j \leq m_2). \end{array}$$

where $L_p^{(k)}(y)$ denotes the k-th localizing matrix of a poly p and a moment vector y.

Flat Truncation (FT)

Suppose y^* is a minimizer of the dual optimization problem:

$$\begin{array}{rll} \min & f^T y & s.t.\ L_h^{(k)}(y) = 0\,(1 \le i \le m_1),\ y_0 = 1, \\ & & L_{g_j}^{(k)}(y) \succeq 0\,(0 \le j \le m_2). \end{array}$$

We say y^* has a flat truncation (FT) or FT holds at y^* if

$$\operatorname{rank} M_t(y^*) \quad = \quad \operatorname{rank} M_{t-d}(y^*) \qquad \text{for some } t \in [d,k]$$

where $d = \max\{1, \lceil \deg(h)/2 \rceil, \lceil \deg(g)/2 \rceil\}$.

FT $\Rightarrow$ $y^*|_{2t}$ admits a finite measure (Curto-Fialkow).

FT $\Rightarrow$ global minimizers can be found (Henrion-Lasserre).

FT Holds Generally

Suppose y^* is a minimizer of the k-th Lasserre's relax.:

$$\min \quad \sum f^T y \quad s.t. \;\; L_{h_i}^{(k)}(y) = 0 \, (1 \le i \le m_1), \, y_0 = 1, \\ L_{g_j}^{(k)}(y) \succeq 0 \, (0 \le j \le m_2).$$

Theorem (N. 2011) If the optimization problem

$$\begin{array}{ll} \min & f(x) \\ s.t. & (h_1, \ldots, h_{m_1})|_x = 0, \, (g_1, \ldots, g_{m_2})|_x \ge 0. \end{array}$$

has finitely many global KKT points, then FT holds for every minimizer y^* for some order k.

Generically, FT can be used as a certificate to check finite convergence of Lasserre's hierarchy.

The main conclusion of this talk

For generic cases, Lasserre's hierarchy has finite convergence.

What can we do for nongeneric cases?

Apply Jacobian SDP relaxation! It always has finite convergence if only the feasible sets are smooth, no matter what the objectives are.

Thank you very much for attention!