Mathematical modelling of sap flow in maple trees

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Maple syrup is a traditional agricultural crop that is of significant importance in eastern Canada and the northeastern United States. Sugarbush owners have amassed a great deal of practical experience in maple sap harvesting techniques, but are increasingly looking to scientists to assist them with developing optimal harvesting strategies as well as in understanding the effects of recent changes in climate on sap yields. Mathematics in particular has a role to play in dealing with the complex coupled physics that govern sap flow in trees.

I will focus in this talk on two mathematical modelling efforts currently underway at SFU. The first aims to understand the phenomenon of *sap exudation*, in which sugar maple and other related species generate positive pressures that initiate sap flow during the spring thaw as temperatures begin to oscillate around the freezing point. Many attempts have been made over the past century to explain this phenomenon by means of different physical and biological mechanisms, but there remains a significant degree of controversy over the precise mechanism driving sap exudation. We consider a prevailing hypothesis due to Milburn–O'Malley and Tyree that treats sap as a two-phase (gas + liquid) mixture whose dynamics are governed by the combined effects of porous transport, sap thawing, gas dissolution and osmotic pressure. A nonlinear system of differential equations is developed that captures these effects at the cellular scale, and we demonstrate through a combination of analytical and numerical methods that the model is capable of reproducing qualitatively many of the behaviours observed in maple trees. The results offer insight into the dynamics of sap exudation, as well as having application to the study of embolism recovery which is a related phenomenon affecting a much larger number of tree species.

The second project is aimed at developing a macroscopic model for sap flow and heat transport in a tree during the growing season when sap flow is driven by the process of *transpiration*. The tree is treated as an anisotropic porous medium through which sap flow is driven by a given transpiration flux, and heat transport is driven by daily variations in ambient temperature and solar radiation. Our current efforts are focused on adapting this model to early spring conditions in maple trees and up-scaling the cellular-level exudation model to obtain temperature-dependent transport coefficients that include freeze/thaw, dissolution and osmotic effects. Our ultimate goal is to develop an whole-tree simulation tool that is efficient enough to perform optimization and parametric studies.

In closing, this talk provides a concrete example of a problem with both industrial applications that has not only given rise to novel mathematical and computational research, but has also contributed significantly to the solution of outstanding problems in plant physiology. This is joint work with Maurizio Ceseri and Bebart Janbek (both at SFU) and is funded by research grants from NSERC, Mitacs and the North American Maple Syrup Council.