Faithful actions of the absolute Galois group on moduli spaces and change of fundamental group

Abstract:

In the 60's J. P. Serre showed that there exists a field automorphism $\sigma \in \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$, and a variety X defined over $\bar{\mathbb{Q}}$ such that X and the Galois conjugate variety X^σ have non isomorphic fundamental groups, in particular they are not homeomorphic. In a joint paper with F. Catanese and F. Grunewald we give a strong sharpening of this phenomenon discovered by Serre:

Theorem 0.1. *If $\sigma \in \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q}))$ is not in the conjugacy class of the complex conjugation then there exists a surface (isogenous to a product) X such that X and the Galois conjugate variety X^σ have non isomorphic fundamental groups.*

Moreover, we give some faithful actions of the absolute Galois group $\mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$, related among them, in particular,

Theorem 0.2. *The absolute Galois group $\mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$ acts faithfully on the set of connected components of the (coarse) moduli spaces of surfaces of general type.*

We will explain the idea of the proof of the second theorem as well as the technical difficulties. Finally we will show how the first theorem can be deduced from the second.