Adiabatic theory in open quantum systems II

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Disclaimer

These slides cover only the second part of my talk (with the exception of adiabatic theorems for stochastic quantum equations). The overview of the field that I presented in the first part is not included.

Outline

Driven Lindblad equation

$$\varepsilon \dot{\rho}(s) = \mathcal{L}(s)\rho(s) \qquad (\varepsilon \to 0)\,,$$

$$\mathcal{L}(s)\rho = -i[H(s), \rho] + \sum_{\alpha} \big(2\Gamma_\alpha(s)\rho\Gamma_\alpha^*(s) - \{\Gamma_\alpha^*(s)\Gamma_\alpha(s), \rho\}\big).$$

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- Solve the equation
- History dependent term and optimization
- Instantaneous term and linear response

Solving the equation $\varepsilon\dot{\rho} = \mathcal{L}\rho$

Expansion close to stationary states $\sigma \in \mathrm{Ker}\mathcal{L}$,

$$\mathrm{Ker}\mathcal{L} = \mathrm{Span}\{\sigma_1, \sigma_2, \ldots \sigma_n\}.$$

The projection into stationary states is

$$\mathcal{P}\rho = \sum_j \sigma_j \mathrm{Tr}(M^j \rho),$$

where M^j are determined by conditions $\mathcal{P}^2 = \mathcal{P}$ and $\mathcal{P}\mathcal{L} = 0$. Their exact form is unimportant.

Still solving the equation

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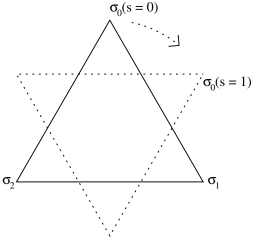

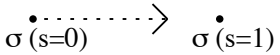

Triangle: Dephasing case, $\sigma_j = P_j$, $M^j = P_j$.
Dot: Generic case, $M = \mathrm{Id}$.

A solution to $\varepsilon\dot{\rho} = \mathcal{L}\rho$

Suppose that 0 is an isolated eigenvalue of $\mathcal{L}$ (Gap condition) and let $\sigma(s)$ be an extremal point of the polytope. Then for $0 \le s \le 1$ the equation has a solution

$$\rho(s) = \sigma(s) + \varepsilon\mathcal{L}^{-1}(s)\dot{\sigma}(s) + \varepsilon\sum_j \sigma_j(s)\int_0^s \mathrm{Tr}\left(\dot{X}^j(\tau)\mathcal{L}^{-1}(\tau)\dot{\sigma}(\tau)\right)\mathrm{d}\tau + o(\varepsilon).$$

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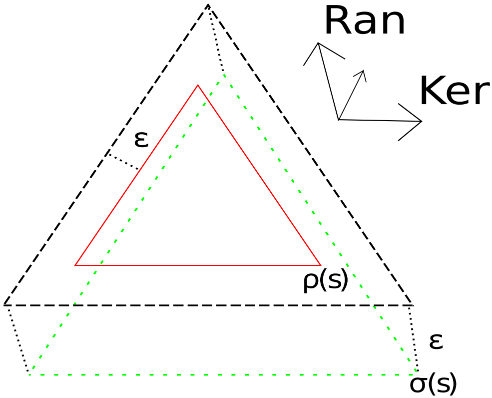
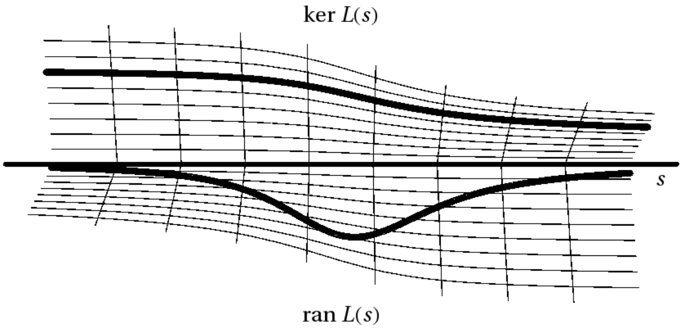
0th order term: Parallel transport inside stationary manifold.
1th order terms: Instantaneous response and history dependent term .

The solution in a picture



Green: Stationary states; Black: Transversal instantaneous correction; Red: Full 1.order correction.

Reversibility in picture



History dependent term for dephasing

Dephasing Lindbladians:
For a given Hamiltonian $H = \sum_j E_j P_j$ a dephasing Lindbladian satisfies

$$\mathcal{L} P_j = 0, \qquad \sigma_j = P_j.$$

It follows $\Gamma_\alpha = \Gamma_\alpha(H)$.

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Stationary states form a polytope when eigenvalues are non-degenerate, $P_j = |j\rangle\langle j|$, then

$$\mathcal{L}|0\rangle\langle j| = (-\gamma_j + i\lambda_j)|0\rangle\langle j|, \quad \gamma_j \geq 0.$$

The projection on the kernel has a form

$$\mathcal{P}\rho = \sum_j P_j \mathrm{Tr}(P_j \rho).$$

History dependent term for dephasing

For a driven Hamiltonian $H(s) = \sum_j E_j(s)P_j(s)$ and a dephasing Lindbladian $\mathcal{L}(s)$ the equation $\varepsilon\dot{\rho} = \mathcal{L}\rho$ has a solution

$$\rho(s) = P_0(s) + \varepsilon \cdots \\ + \varepsilon \sum_{j\neq 0} (P_j(s) - P_0(s)) \int_0^s R_j(\tau)\,\mathrm{d}\tau,$$

where a tunneling rate

$$R_j(\tau) = 2\frac{\gamma_j(\tau)}{\lambda_j^2(\tau) + \gamma_j^2(\tau)} \mathrm{Tr}(P_0(\tau)\dot{P}_j^2(\tau)P_0(\tau)) \geq 0.$$

Optimization of adiabatic control

Given a time ε^{-1} and path $H(s)$ find a re-parametrization $s(\varphi)$ such that the evolution $\varepsilon\dot{\rho}(s) = \mathcal{L}(s)\rho(s)$ minimize tunneling

$$T = 1 - \mathrm{Tr}(\rho(1)P_0(1)) = \varepsilon \int_0^1 R(s)\dot{s}^2 \,\mathrm{d}\varphi.$$

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Solution

$$\dot{s}^2 \sim \frac{1}{R(s)} \sim \frac{\lambda^2(s) + \gamma^2(s)}{\gamma(s)} \frac{1}{\mathrm{Tr}(P_0(s)\dot{P}_1^2(s)P_0(s))}.$$

- The control is local: "Keep constant tunneling".
- No prior knowledge of the system is required.

Linear response theory

We study response of an observable X to a driving $\phi(s)$,
$\dot{\rho} = \mathcal{L}(\phi)\rho$,

$$\mathrm{Tr}(X\rho(s)) = \cdots + \#\,\dot{\phi} + o(\dot{\phi}).$$

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Response of fluxes $\dot{X}$,

$$\begin{aligned}
\mathrm{Tr}(\dot{X}\rho) &= \mathrm{Tr}\left[\mathcal{L}^*(X)\left(\sigma + \mathcal{L}^{-1}\partial\sigma\dot{\phi} + \mathit{Hist.T.} + o(\dot{\phi})\right)\right] \\
&= \mathrm{Tr}\left[X\mathcal{L}\left(\sigma + \mathcal{L}^{-1}\partial\sigma\dot{\phi} + \mathit{Hist.T.} + o(\dot{\phi})\right)\right] \\
&= \mathrm{Tr}[X\partial\sigma]\dot{\phi} + o(\dot{\phi}).
\end{aligned}$$

- Response of fluxes depends on stationary states, not on dynamics!

Response of Isospectral family

Consider symmetry generators G_ν,

$$H(\phi^\nu) = \exp(iG_\nu \phi^\nu) H \exp(-iG_\nu \phi^\nu),$$
$$\Gamma_\alpha(\phi^\nu) = \exp(iG_\nu \phi^\nu) \Gamma_\alpha \exp(-iG_\nu \phi^\nu).$$

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Then the response of $\dot{G}_\nu$ is geometric, given by

$$\begin{aligned}\mathrm{Tr}(\dot{G}_\mu\rho) &= \mathrm{Tr}(G_\mu\partial_\nu\sigma)\dot{\phi}^\nu + o(\dot{\phi}) \\ &= \mathrm{Tr}(G_\mu i[G_\nu,\sigma])\dot{\phi}^\nu + o(\dot{\phi}) \\ &= -i\mathrm{Tr}([G_\mu,\ G_\nu]\sigma)\dot{\phi}^\nu + o(\dot{\phi}).\end{aligned}$$

Example: When G_μ are shears on a plane, then $[G_\mu, G_\nu]$ is a rotation and the formula relates Hall viscosity to the angular momentum.

Conclusions, outlooks

- Adiabatic expansion of the equation $\varepsilon\dot{\rho}(s) = \mathcal{L}(s)\rho(s)$ near stationary states.
- Driven dephasing Lindbladians and their application to optimal control.
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- Counting of resources in open system.
- Response of open many body systems.

Thanks for your attention!

References

- Adiabatic expansion: Avron et. al., arXiv:1106.4661
  M. S. Sarandy, D. A. Lidar, arXiv:quant-ph/0404147v4
- Optimal control: Avron et. al., arXiv:1003.2172
  See also Rezakhani et. al., arXiv:0905.2376
- Linear response: Avron et. al., arXiv:1202.5750
  Example with Shears: N. Read, E. H. Rezayi, arXiv:1008.0210.