Local Thermal States and Correlations

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Information theoretic approaches to thermodynamics
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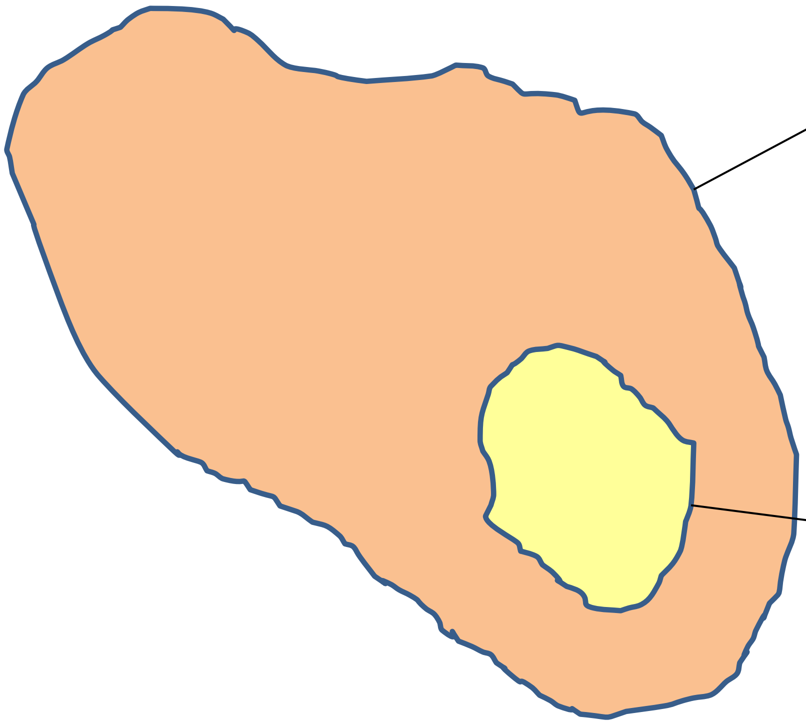
Local Temperature



The whole system has "thermalized" and the global state is thermal:

$$\rho_S = \frac{\exp(-\beta H_S)}{Z} = \Omega_S$$

Reduced state of the block:

$$\rho_B = \mathrm{tr}_{S-B}(\Omega_S)$$

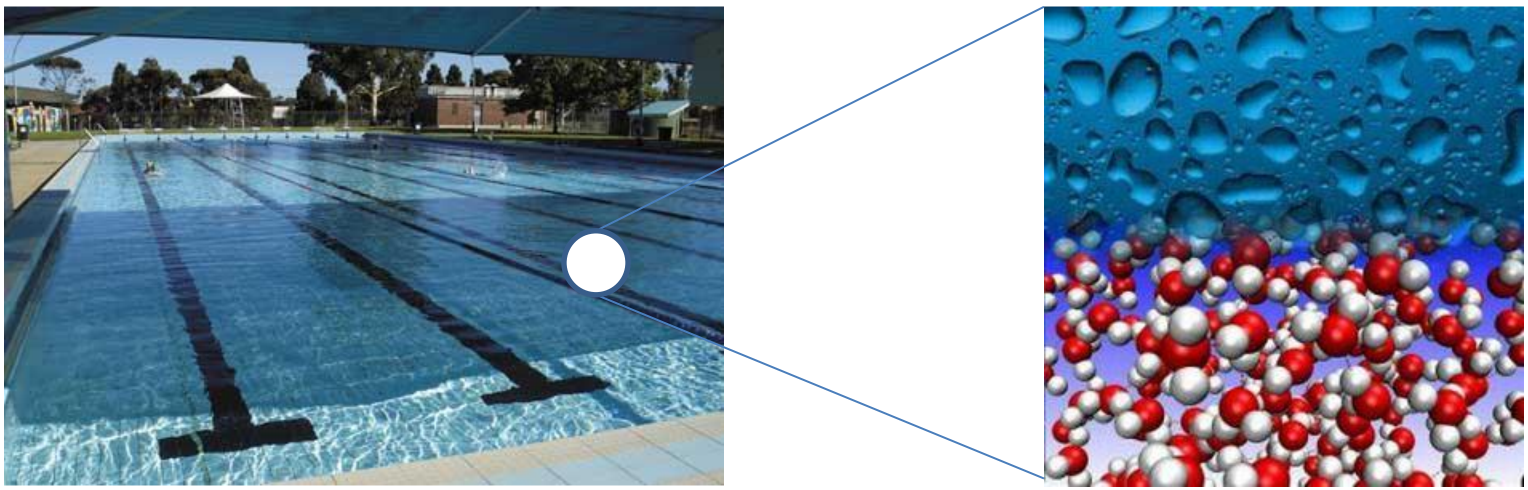
Is the block in a thermal state at the same temperature as the whole system?

Motivations

- What are typical states of thermalized systems?
- Is temperature an intensive quantity?
- How can temperature be estimated from local observables?
- Which role do correlations play in the definition of temperature?
- Which correlations? Classical or quantum?

Some of these issues have previously been considered in:
M. Hartmann, G. Mahler and O. Hess, Phys. Rev. Lett. 93, 080402 (2004).

Thermodynamics: a huge simplification



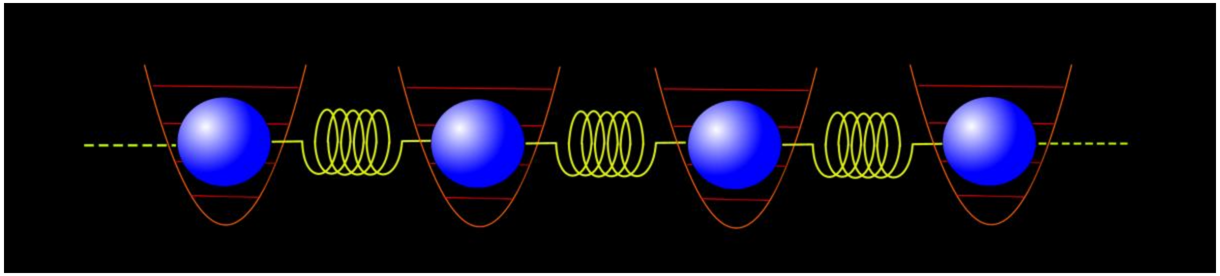
Temperature: 25^{o} C
Length: 50 m

N_a molecules of H_2O

This simplification comes from the fact that we have access to coarse grained observables.

The model and the tools

We consider systems composed of coupled harmonic oscillators in 1D and 2D geometries.



$$H = \frac{1}{2}\sum_i p_i^2 + \sum_{i,j} q_i V_{ij} q_j \longrightarrow$$ Coupling in the position degrees of freedom

Chain with nearest-neighbor interactions: $V = \mathrm{circ}(1, -c, 0, \ldots, 0, -c)$

Criticality: $c \to \frac{1}{2^D}$

Advantages

- Ground and thermal states are Gaussian → large-size computations become possible → "thermodynamical" limit.
- These systems model several interesting physical systems: crystal lattices, trapped ions, nano-mechanical oscillators.
- Quantum information quantities can be efficiently calculated for Gaussian states: entanglement, correlation measures, fidelities, partial traces,…

Gaussian thermal states

$$H = \frac{1}{2}\sum_i p_i^2 + \sum_{i,j} q_i V_{ij} q_j$$

- Ground state:

$$\gamma_{\mathrm{gr}} = V^{-1/2} \oplus V^{1/2}$$

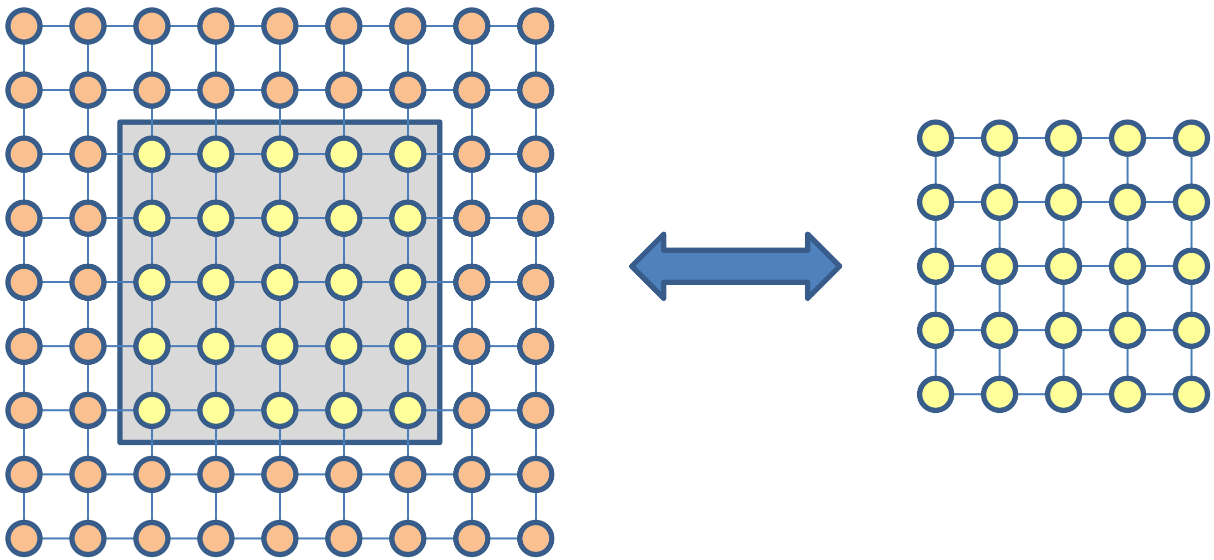
- Thermal state:

$$\gamma_{\mathrm{th}} = V^{-1/2} W(\beta) \oplus V^{1/2} W(\beta)$$

with

$$W(\beta) = 1 + 2\left(\exp\left(\beta V^{1/2}\right) - 1\right)^{-1}$$

Statement of the problem



$$\rho_B = \mathrm{tr}_{S-B}(\Omega_S) \qquad\qquad \Omega_B = \frac{\exp(-\beta H_B)}{Z}$$

Similarity between these two states: $F(\rho_B, \Omega_B)$

Distinguishability over all possible measurements.

Temperature is intensive whenever $F \approx 1$

Hamiltonian for the block

A temperature-independent Hamiltonian for the block is necessary.

Criterion: in the limit of high temperatures, intensiveness should be recovered, $F \to 1$.

$$\gamma \xrightarrow[\beta \to 0]{} \frac{2}{\beta}\left(V^{-1} \oplus 1\right) \qquad \gamma_B \xrightarrow[\beta \to 0]{} \frac{2}{\beta}\left(\tilde{V}^{-1} \oplus 1\right)$$

$$\tilde{V} = V_B - V_{BR} V_R^{-1} V_{BR}^T \qquad V = \begin{pmatrix} V_B & V_{BR} \\ V_{BR}^T & V_R \end{pmatrix}$$

Putting these two things together:

$$H_B = \frac{1}{2} \sum_i p_i^2 + \sum_{i,j} q_i \tilde{V}_{ij} q_j$$

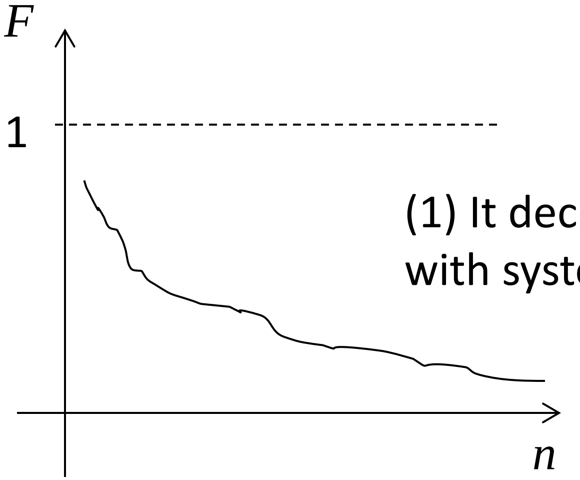
Hamiltonian for the block

- Other criteria lead to the same Hamiltonian for the block.

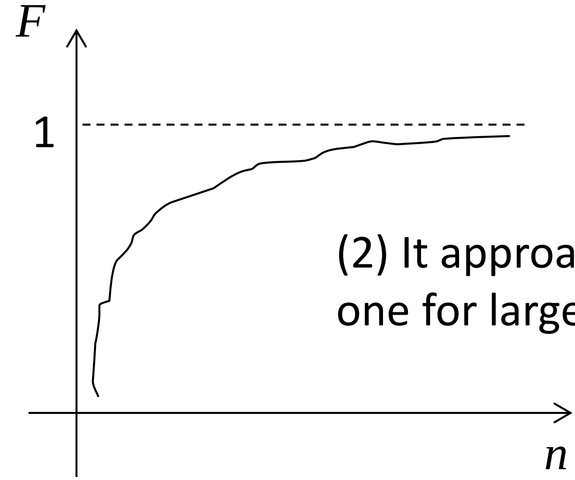
- In the case of coupled classical harmonic oscillators, this Hamiltonian leads to an intensive fidelity for all values of the temperature and system sizes.

- This choice makes F equal to one in the large-T limit.

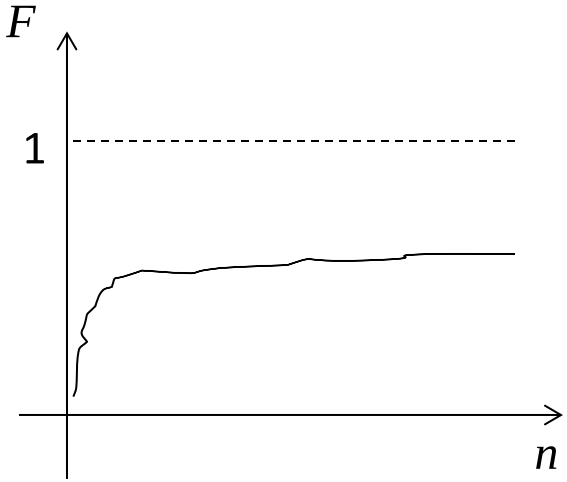
Guess the fidelity behavior!



(1) It decreases with system size.

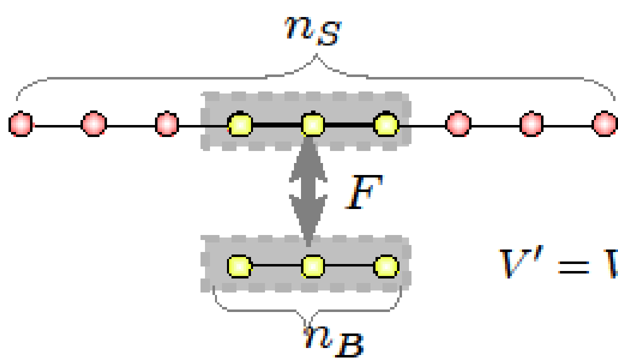


(2) It approaches one for large sizes.



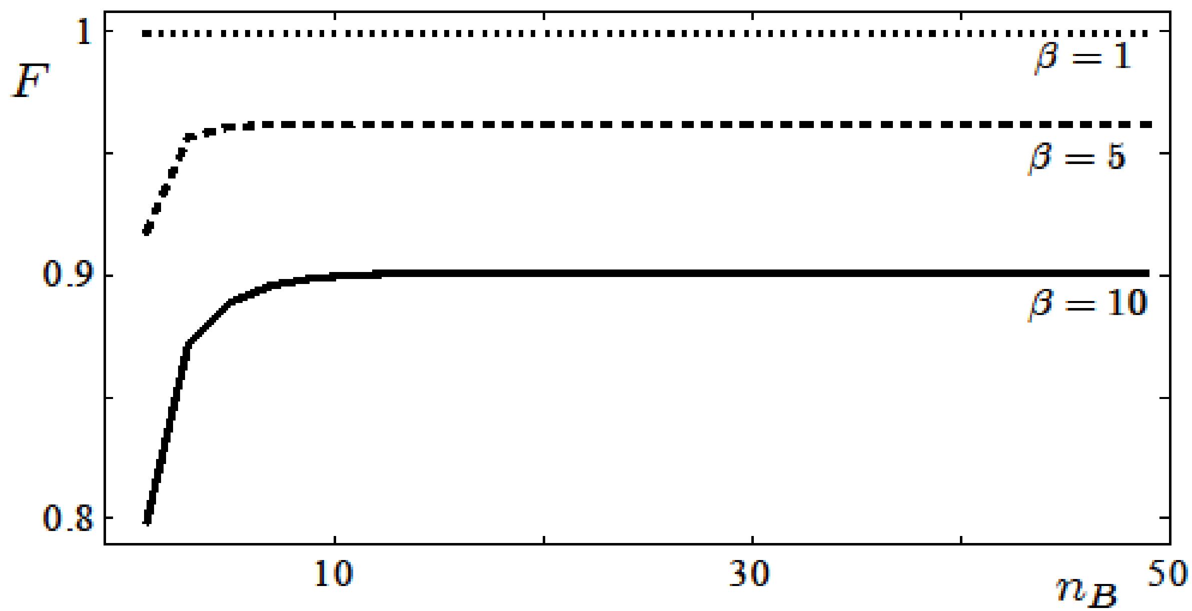
(3) It saturates, though <1, for large sizes.

Results: 1D – the fidelity saturates



$$V_1 = \mathrm{circ}(1, -c, 0, ..., 0, -c)$$

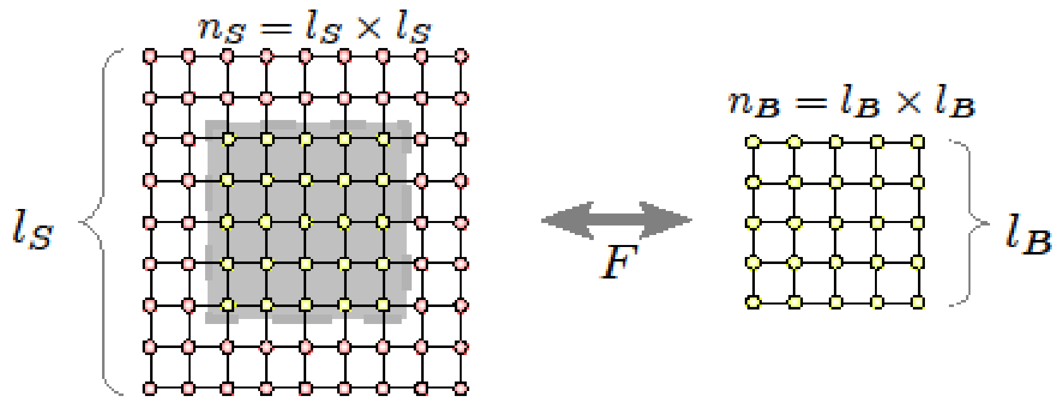
$$V' = V_B - V_{BR} V_R^{-1} V_{BR}^T$$



$n_S = 400$
$c = 0.4999$

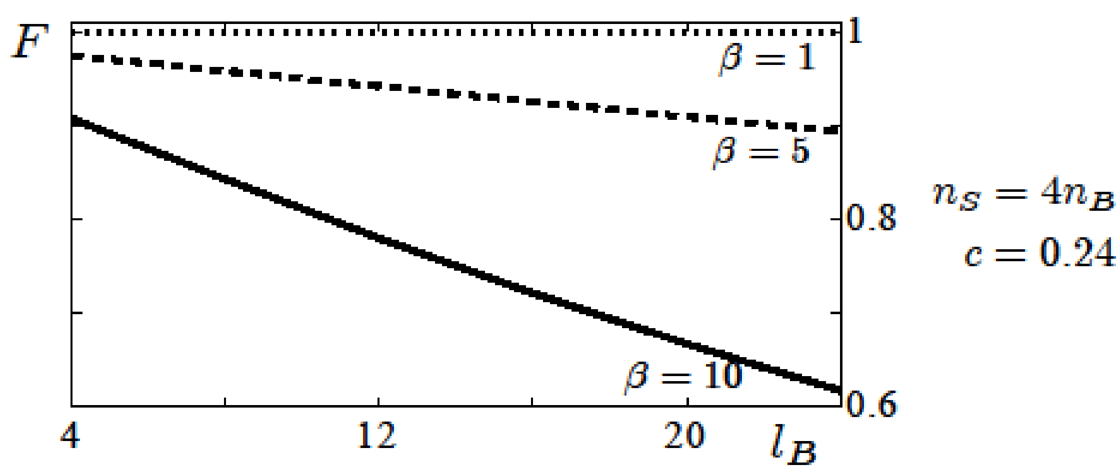
The fidelity follows an area law

Results: 2D – the fidelity decreases



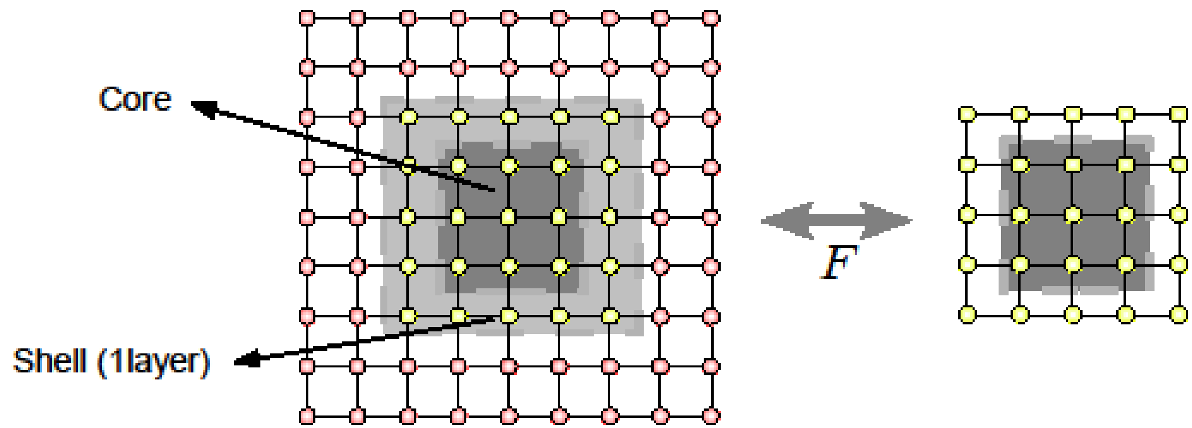
Larger deviations from intensiveness for larger systems!

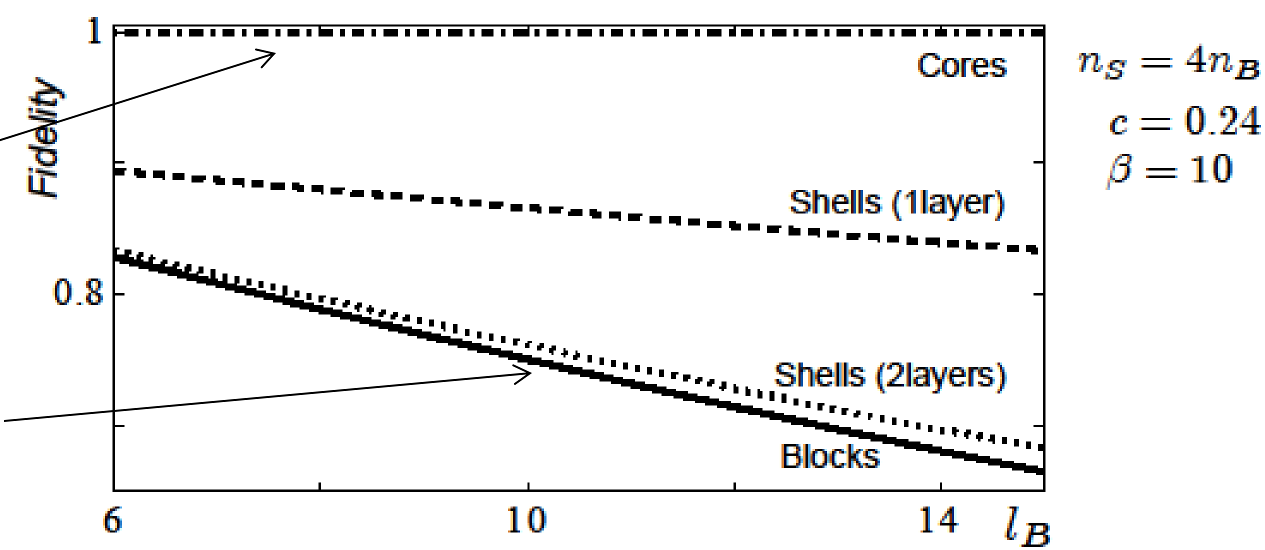
(despite the more and more negligible weight of the interaction at the boundary)



The fidelity follows an area law

Deviation from intensiveness resides in the boundary





The cores are effectively indistinguishable.

All the distinguishability is at the boundary.

Local Thermal States

- The distinguishability between the ideal thermal state and the actual state increases with the system and block size.

- Still, the two states become indistinguishable for standard coarse grained observables. Example: magnetization.

$$M = \sum_i \sigma_z^i$$

$$\mathrm{tr}(M\,\rho_B) = \sum_i \mathrm{tr}\left(\sigma_z^i \rho_B^i\right) \approx \sum_{i \in \mathrm{core}} \mathrm{tr}\left(\sigma_z^i \rho_B^i\right) \approx \mathrm{tr}(M\,\Omega_B)$$

- It is possible to define an effective thermal state already for small sizes:

$$\rho_B \approx \mathrm{tr}_\varepsilon\left(\Omega_{B+\varepsilon}\right)$$

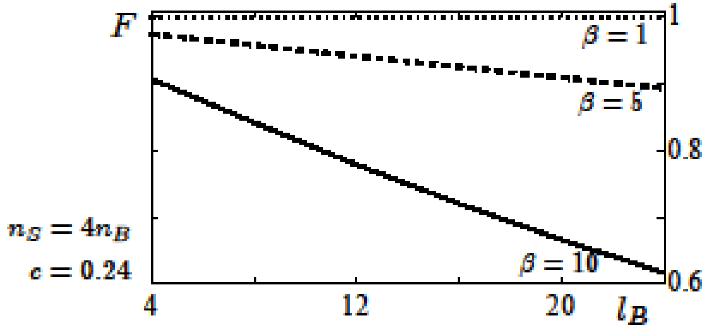
Classical and quantum correlations

Since the thermal state is mixed, it contains all forms of correlations.

- **Total correlations:**
  1. Mutual Information
  2. Correlation Length
- **Quantum correlations:**
  1. Entanglement
  2. Quantum discord

Does any of these correlations play a role in this discussion?

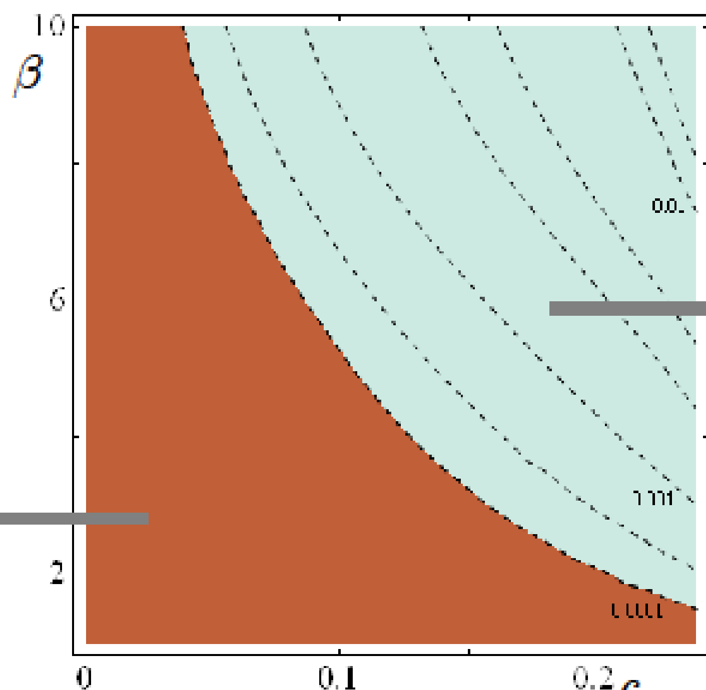
Temperature remains intensive in relevant cases



$$F \approx \alpha_F l_B$$

Size independent evaluation of intensiveness breakdown

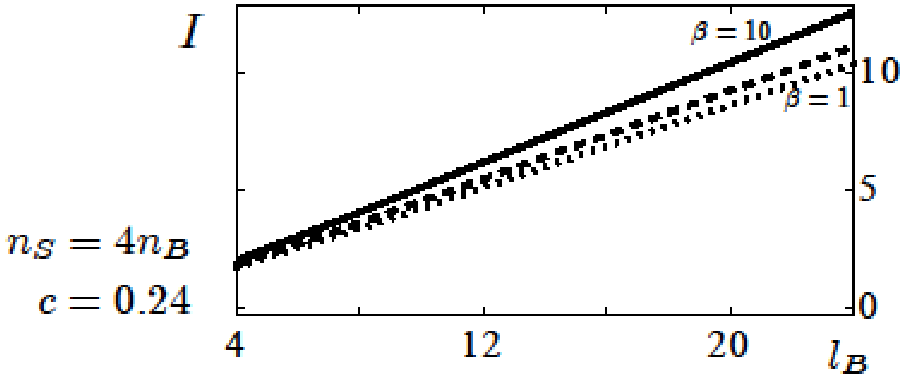
Contour plot of α_F



T not intensive

T intensive

Mutual Information

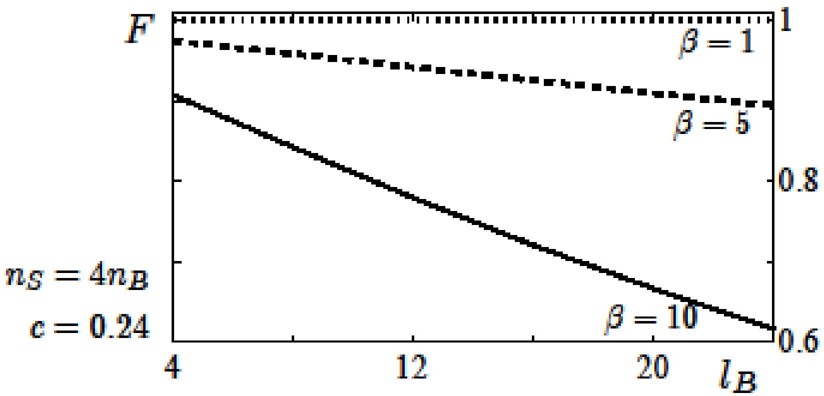


MI follows an area law, however MI is finite for high temperatures

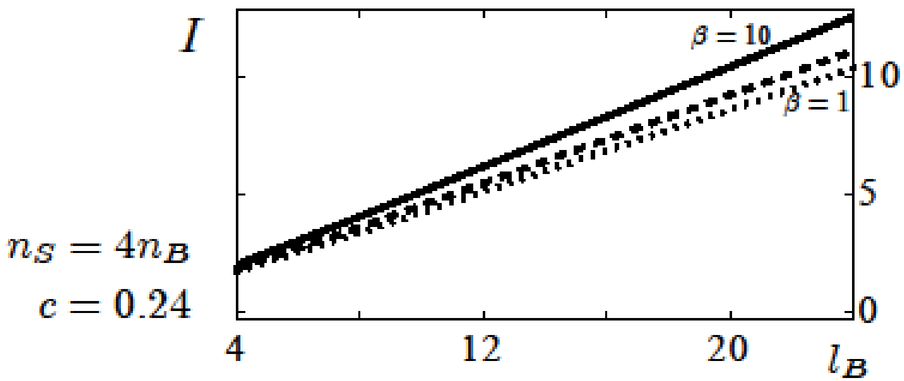
$$I \approx \alpha_I l_B$$

Size independent slope

Compare with the Fidelity



Mutual Information

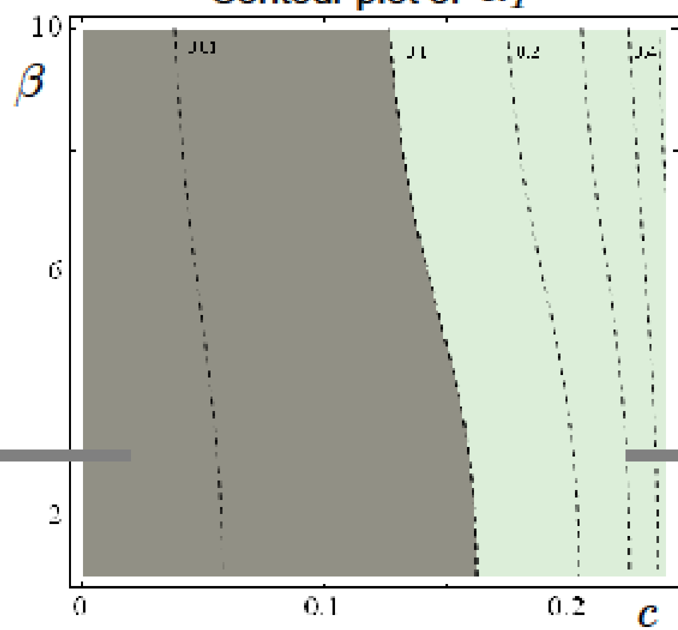


MI follows an area law, however MI is finite for high temperatures

$$I \approx \alpha_I l_B$$

Size independent slope

Contour plot of α_I



MI fails in detecting the intensiveness region

(MI is rather unaffected by T)

"Small" MI

Large MI

Correlation Length

Two-point correlation function (nearest-neighbors):

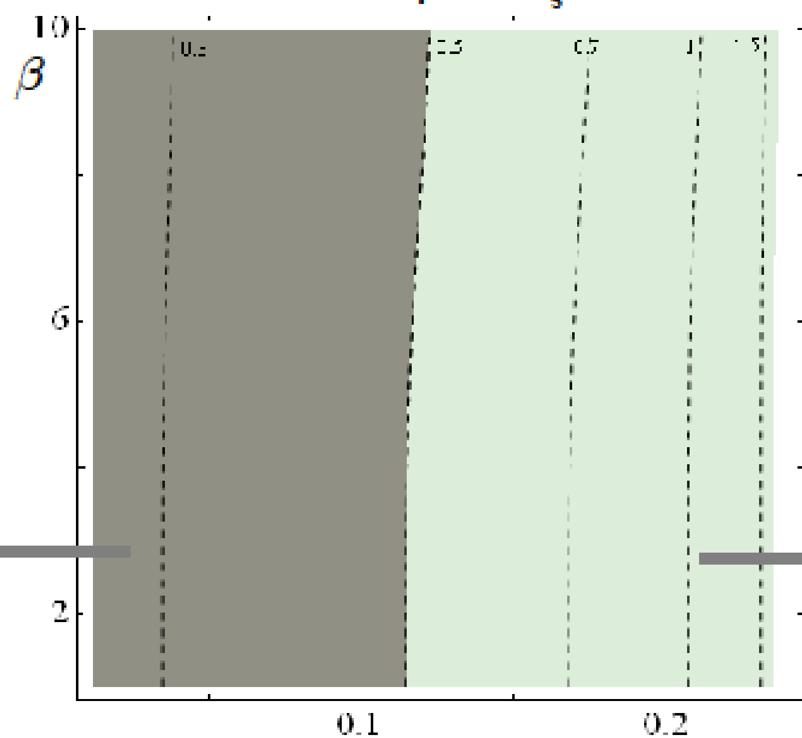
$$\langle q_i q_{i+r} \rangle \propto e^{-\frac{r}{\xi}}$$

ξ defines the correlation length

The correlation length fails in detecting the intensiveness region

(it is rather unaffected by T)

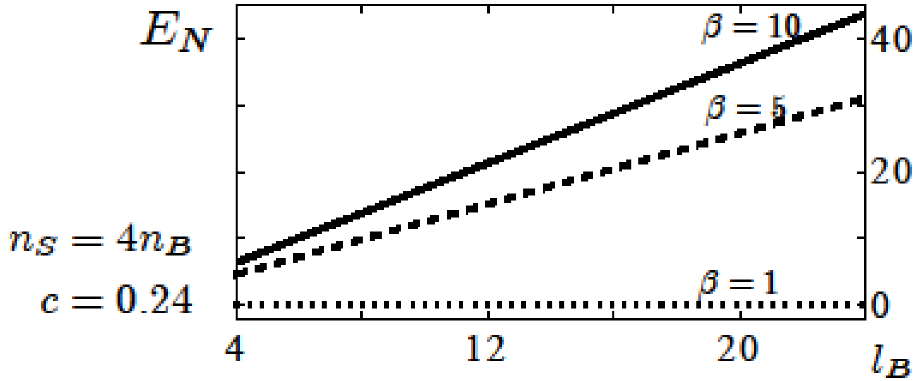
Contour plot of ξ



Small ξ

Large ξ

Log-Negativity of Entanglement

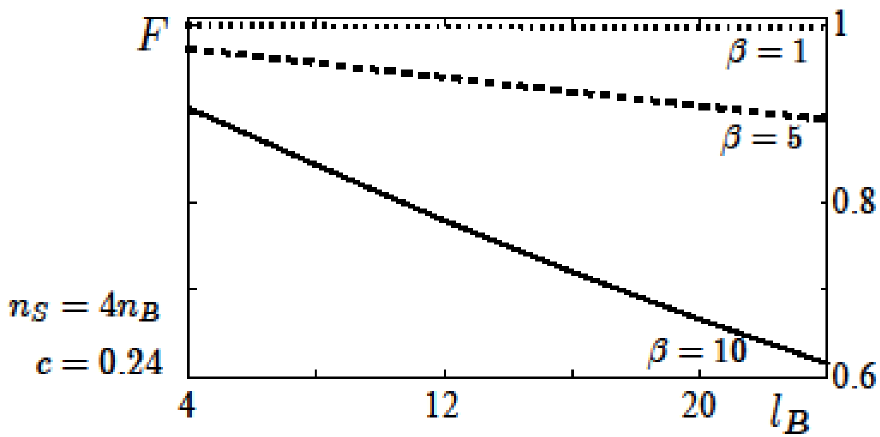


E_N follows an area law and it is zero for high T

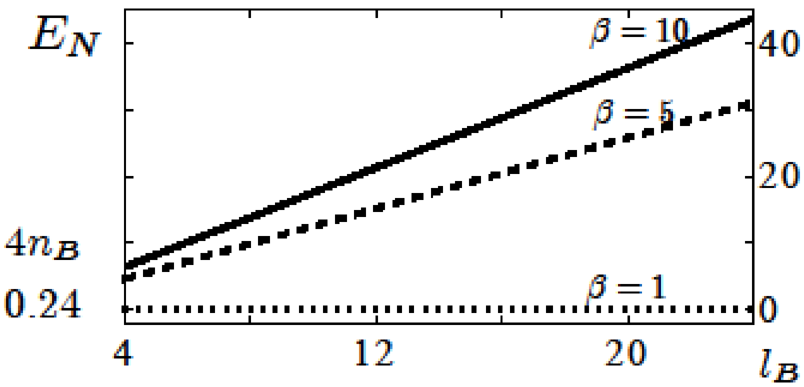
$$E_N \approx \alpha_E l_B$$

Size independent slope

Compare with the Fidelity



Log-Negativity of Entanglement

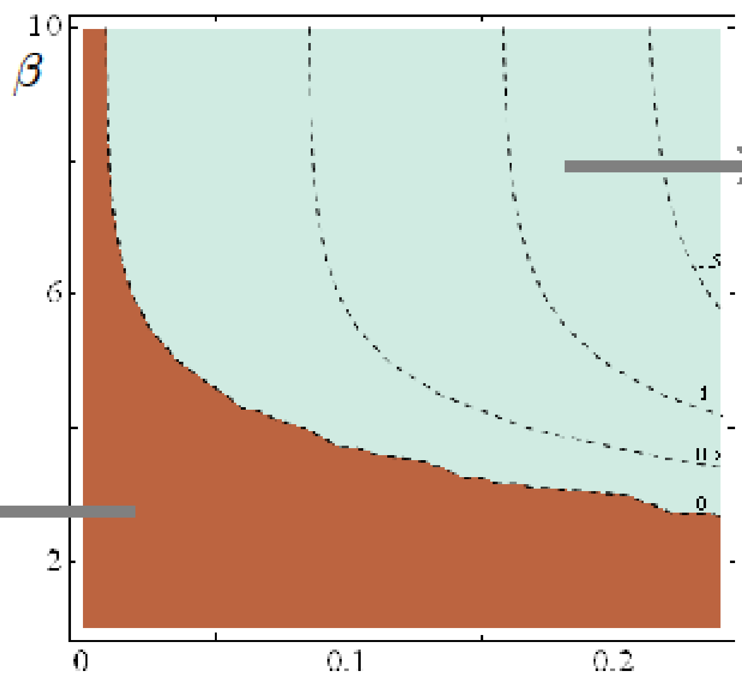


E_N follows an area law and it is zero for high T

$$E_N \approx \alpha_E l_B$$

Size independent slope

Contour plot of α_E



Large amount of Entanglement

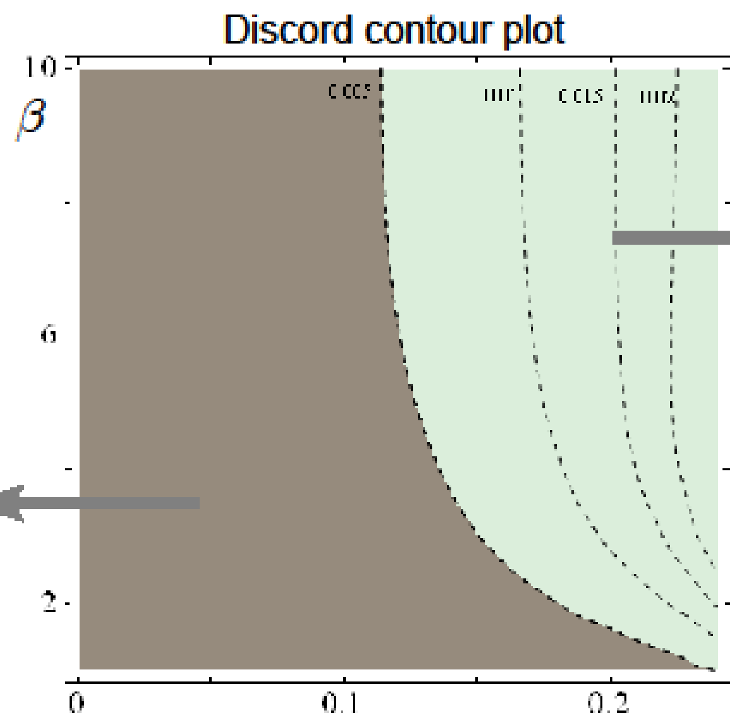
Entanglement detects the intensiveness crossover

Zero Entanglement

Quantum Discord

Gaussian quantum discord between nearest-neighbors
[G.Adesso *et al.*, arXiv:1003.4979; P.Giorda *et al.*, arXiv:1003.3207]:

Also the discord detects the intensiveness crossover



Large Discord

Small Discord

Summary of results

- The distinguishability between the ideal thermal state and the actual state follows an area law.
- For “standard” observables, the two states become more and more undistinguishable.
- An effective thermal state can be associated to the block for any size, temperature and observables.
- Entanglement seems to capture these effects.

Further work

- Extension to other systems. We have also obtained similar results, namely area law for the fidelity, for spin-one-half systems. Still, more calculations are needed.
- Analytical results? Locality!
- Strengthen the connection with quantum correlations (if any).
- Extension to dynamical processes.
- Relation to experiments?