Relating and contrasting plain and prefix Kolmogorov complexity

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We present a short proof of Solovay's results from [5] relating plain and prefix complexity:

$$
\begin{aligned}
K(x) &= C(x) + CC(x) + O(CCC(x)) \\
C(x) &= K(x) - KK(x) + O(KKK(x)),
\end{aligned}
$$

(here $CC(x)$ denotes $C(C(x))$, etc.).

In [3] a short proof is given that some strings have maximal plain Kolmogorov complexity but not maximal prefix-free complexity. The same proof technique was applied to solve three open problems (see [2]):

1) There exist a sequence ω such that $\liminf C(\omega_1 \ldots \omega_n) - C(n)$ is infinite and $\liminf K(\omega_1 \ldots \omega_n) - K(n)$ is finite, i.e. the infinitely often C-trivial reals are not the same as the infinitely often K-trivial reals, (i.e. [1, Question 1]).

2) The 2-random sequences are exactly those sequences that have infinitely many initial segments with maximal plain complexity. This also holds for prefix complexity. We show that some initial segments of a 2-random sequence with maximal plain do not have maximal prefix complexity (a question from L. Bienvenu).

3) We show that there exist no monotone relation between probability and expectation bounded randomness deficiency, (i.e. [4, Question 1]). Because of its simplicity, we present this proof.

If time permits, we discuss an unrelated negative result. Van Lambalgen theorem is shown for Martin-Löf randomness relative to computable measures: a pair of sequences (α, β) is random if and only if α is random and β is random relative to α. We argue that the theorem can not be generalized for randomness relative to a computable measure (a question from A. Shen and H. Takahashi [6]).

References

[1] G. Barmpalias. Algorithmic randomness and measures of complexity. *The Bulletin of Symbolic Logic*, 19(3), Sept. 2013.

[2] B. Bauwens. Relating and contrasting plain and prefix Kolmogorov complexity. *ArXiv e-prints*, November 2013.

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[3] B. Bauwens and A. Shen. Complexity of complexity and maximal plain versus prefix-free Kolmogorov complexity. *Journal of Symbolic Logic*, 2013. Accepted for publication.

[4] L. Bienvenu, P. Gács, M. Hoyrup, C. Rojas, and A. Shen. Algorithmic tests and randomness with respect to a class of measures. *Proceedings of the Steklov Institute of Mathematics*, 274(1):34–89, 2011.

[5] R.M. Solovay. Draft of a paper (or series of papers) on Chaitin's work. 215 pp., unpublished, May 1975.

[6] Hayato Takahashi. Generalization of van lambalgen's theorem and blind randomness for conditional probability. *arXiv preprint arXiv:1310.0709*, 2013. Presented in sept 2013 at CCR in Moscow.