An effective form of Rademacher's theorem

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Abstract

Over the last several years, numerous results relating differentiability of computable functions and algorithmic randomness have been published. Most of them are concerned with functions of one variable. Analysis in higher dimensions is often considerably more challenging and relatively few results connecting differentiability and randomness are known for functions of several variables.

This talk is about two such results. First, I will discuss an effective version of Rademacher's theorem. Rademacher's theorem states that every Lipschitz function $f : \mathbb{R}^n \to \mathbb{R}^m$ is almost everywhere differentiable. An effective form of that theorem states that computable randomness is sufficient for differentiability of computable Lipschitz functions of several variables. Second, I will present a characterisation of weak 2-randomness in terms of differentiability of computable functions in higher dimensions.