Randomness for capacities with applications to random closed sets

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I will prove the following conjecture of Diamondstone and Kjos-Hanssen: the members of the Martin-Löf random closed sets (of Barmpalias, Brodhead, Cenzer, Dashti, and Weber) are exactly the Martin-Löf randoms of measures with finite $\log_2(3/2)$-energy (a concept from potential theory). I will also present similar results for some of the random sets associated with Brownian motion.

These results are applications of a more general theory, namely that Martin-Löf randomness can be extended to subadditive measures called capacities. Capacity theory is an important part of both potential theory in analysis and the theory of random sets in probability. Capacity theory provides a unified framework to study a number of topics in algorithmic randomness—including strong s-randomness, s-energy randomness, algorithmically random closed sets, effective Hausdorff dimension, randomness for classes of measures, and randomness for semimeasures. Moreover, capacity theory—which has been throughly investigated over the last 60 years—provides a wealth of classical results to draw upon to prove new results in algorithmic randomness.