Defining totality in the enumeration degrees

Abstract

The Turing degrees embed naturally into the enumeration degrees. The total degrees are the image of the Turing degrees under this embedding. We prove that the total degrees are definable in the enumeration degrees (in the language of partial orders). This builds on work of Ganchev and Soskova, who defined the total degrees in the local structure of the enumeration degrees. In fact, we show that the same definition works in the global structure. The definition relies on $\mathcal{K}$-pairs, a generalization of semi-computability that Kalimullin introduced in his definition of the enumeration jump. By characterizing the enumeration degrees of semi-computable sets in terms of maximal $\mathcal{K}$-pairs, we obtain a definition of the total degrees.

This talk describes joint work with Cai, Ganchev, Lempp, and Soskova.