Schnorr-randomness versions of K, C, LR, vL-reducibilities

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One of natural measures of randomness is K*-reducibility*, which is defined by $X \leq_K Y$ if and only if

$$K(X \upharpoonright n) \leq K(Y \upharpoonright n) + O(1).$$

Much effort has been devoted to the study of this and related reducibilities.

One direction to analyse K-reducibility is the one via lowness. In fact, $X \leq_K \emptyset$ if and only if $X \leq_{LR} \emptyset$ if and only if $X \leq_{LK} \emptyset$, which is shown by Nies [8]. Kjos-Hanssen et al. [2] strengthned this result to that LK-reducibility is actually equivalent to LR-reducibility.

Another important reducibility to analyse K-reducibility is *vL-reducibility*. For ML-random sets, vL-reducibility is the converse of LR-reducibility. Miller and Yu [4] showed that $X \leq_K Y$ implies $X \leq_{vL} Y$.

We consider Schnorr-randomness versions of these results. *Schnorr reducibility* is the Schnorr-randomness version of K-reducibility, which is defined by $X \leq_{Sch} Y$ if for every computable measure machine M there is a computable measure machine N such that $K_N(X \upharpoonright n) \leq K_M(Y \upharpoonright n) + O(1)$.

Kjos-Hanssen et al. [3] showed that a set is low for Schnorr randomness if and only if it is computably traceable. Then, Nies [9, Problem 8.4.22] asked whether the reducibility versions are equivalent or not. We answer this affirmatively using the open covering method developed by Bienvenu and Miller [1]. A similar result was obtained in [5] with uniform relativization.

Miyabe [6] and Miyabe and Rute [7] showed van Lambalgen's theorem for uniform Schnorr randomness. Thus, we can consider a Schnorr-randomness version of vL-reducibility. We show that Schnorr reducibility implies the Schnorr-randomness version of vL-reducibility. The key of the proof is an extension of the Ample Excess Lemma.

References

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