ON ROGERS SEMILATTICES OF ANALYTICAL HIERARCHY

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We investigate some algebraic properties of Rogers semilattices of analytical hierarchy: existence of minimal elements, ideals without minimal elements. For an at most coutable non-empty family S of subjects of the natural series, its numbering $\alpha : N \to S$ is said to be Σ^1_{n+1}-computable if the set $\{\langle x, y\rangle \mid x \in \alpha(y)\} \in \Sigma^1_{n+1}$. The set of all Σ^1_{n+1}-computable numberings of the family S is denoted by $Com^1_{n+1}(S)$. Enumeration $\nu \in Com^1_{n+1}(S)$ is called minimal, if for every $\mu \in Com^1_{n+1}(S)$ such that $\mu \leq \nu$, perfomed $\nu \equiv \mu$. One of the most important minimal numberings is Friedbergs numbering. Owings showed in [2] that there is no Π^1_1-computable Friedberg enumeration of all Π^1_1-sets using metarecursion theory. This result is obtained in classic computability theory for higher levels of analytical hierarchy.

Theorem

(1) There are infinitely many minimal numberings of an infinite family S of Π^1_{n+1}-sets.

(2) There is no a Π^1_{n+1}-computable Friedberg enumeration of all Π^1_{n+1}-sets.

(3) Elementary theory of any nontrivial Rogers semilattices of analytical hierarchy is undecideble.

(4) Let S be infinite family of Σ^1_{n+1}-sets, $Com^1_{n+1}(S) \neq \emptyset$. Then there exists a numbering $\beta \in Com^1_{n+1}(S)$ such that $\hat{\beta}$ (the principal ideal of Rogers semilattices $R^1_{n+1}(S)$ generated by $deg(\beta)) contains no minimal elements$.

This work was supported by RFBR (grant 14-01-31278).

References

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[2] James C. Owings, ***The meta-r.e. sets, but not the Π^1_1-sets can be enumerated without repetition***,The Journal of Symbolic Logic, Volume 35, Number 2, June 1970.

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