FINITE AUTOMORPHISM BASES FOR DEGREE STRUCTURES

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An automorphism base for a structure $\mathcal{A}$ is a subset of its domain B with the following property: if a pair of automorphisms of $\mathcal{A}$ agree on the elements in the set B then they are equal. Slaman and Woodin [4] investigated properties of the automorphism group of the Turing degrees. They show that it is countable and that all its members have an arithmetic presentation. The key to these results lies in their proof that the Turing degrees have a finite automorphism base, in fact every 5-generic element is an automorphism base for $\mathcal{D}_T$.

The work presented in this talk is joint with Theodore Slaman. We first investigate automorphisms of the local substructure of all Turing degrees below $\mathbf{0}'$, denoted by $\mathcal{D}_T(\leq \mathbf{0}')$. We show that $\mathcal{D}_T(\leq \mathbf{0}')$ has a finite automorphism base and thus its automorphism group has similar properties as that of the Turing degrees: it is countable, every member has an arithmetically definable presentation.

Next we consider the structure of the enumeration degrees $\mathcal{D}_e$. It is an upper semi-lattice with least element and jump operation, based on a positive reducibility between sets of natural numbers, enumeration reducibility, introduced by Friedberg and Rogers [2]. The Turing degrees have a natural isomorphic copy in the structure of the enumeration degrees, namely the substructure of the total enumeration degrees. Selman [3] showed that the total enumeration degrees are an automorphism base for the enumeration degrees. Recently Cai, Ganchev, Lempp, Miller and Soskova [1] proved that the total enumeration degrees are first order definable in $\mathcal{D}_e$. This allows us to transfer the automorphism analysis of $\mathcal{D}_T$ by Slaman and Woodin to the structure of the enumeration degrees: there is a finite automorphism base for $\mathcal{D}_e$, consisting of elements bounded by $\mathbf{0}_e^{(5)}$. Based on this and using further definability properties of the structure $\mathcal{D}_e$ we were able to bring down the level of arithmetic complexity needed to obtain an automorphism base for $\mathcal{D}_e$: we show that there is a finite automorphism base for $\mathcal{D}_e$ consisting of total elements bounded by $\mathbf{0}_e'$.

References

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This research was supported by a BNSF grant No. DMU 03/07/12.12.2011, by a Sofia University SF grant and by a Marie Curie international outgoing fellowship STRIDE (298471) within the 7th European Community Framework Programme.