

The realizability problem as an infinite dimensional moment problem

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Abstract

The realizability problem is an infinite dimensional version of the classical moment problem which naturally arises from applied fields dealing with the analysis of complex systems like many-body systems in statistical mechanics, spatial ecology, stochastic geometry, heterogeneous materials, etc. Since such a system consists of a huge number of identical components, its investigation is greatly facilitated if the attention is restricted to selected characteristics (usually correlation functions) which encode the most relevant properties of the system. The realizability problem exactly addresses the question whether a given candidate correlation function actually represents the correlation function of some random distribution.

In this talk, I present a joint work with T. Kuna and A. Rota about the realizability problem on semi-algebraic sets. In particular, we derive necessary and sufficient conditions for an infinite sequence of generalized functions to be the moment sequence of a measure concentrated on a closed semi-algebraic subset of the space of distributions. Our approach combines the classical results about the moment problem on nuclear spaces with some techniques recently developed to treat the moment problem on basic semi-algebraic sets of \mathbb{R}^d . In this way, we determine realizability conditions that can be more easily verified than the well-known Haviland type conditions, giving a complete characterization of the support of the realizing measure in terms of its moments. Moreover, I introduce some concrete examples of semi-algebraic sets of generalized functions and outline some open problems related to this topic.