COMPUTATIONS OF SOME KRONECKER COEFFICIENTS.

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Let $S(\mathbb{C}^{n_1} \otimes \mathbb{C}^{n_2} \otimes \mathbb{C}^{n_3})$ be the space of polynomials in $n_1 n_2 n_3$ variables. This space carries a natural representation of the group $GL(n_1) \times GL(n_2) \times GL(n_3)$. It is an important representation space, in particular for its relation with quantum computing.

Then

$$S(\mathbb{C}^{n_1} \otimes \mathbb{C}^{n_2} \otimes \mathbb{C}^{n_3}) = \oplus g(\lambda, \mu, \nu) V_\lambda^{GL(n_1)} \otimes V_\mu^{GL(n_2)} \otimes V_\nu^{GL(n_3)}$$

is a direct sum of the irreducible representations $V_\lambda^{GL(n_1)} \otimes V_\mu^{GL(n_2)} \otimes V_\nu^{GL(n_3)}$ of the group $GL(n_1) \times GL(n_2) \times GL(n_3)$, with multiplicities $g(\lambda, \mu, \nu)$. Coefficients $g(\lambda, \mu, \nu)$ are related with Kronecker coefficients. The function $t \to g(t\lambda, t\mu, t\nu)$ is a quasi polynomial function of t (here t varies over the non negative integers). This function generalizes the function $t- > c(t\lambda, t\mu, t\nu)$ for dilated Clebsch-Gordan coefficients.

We present an algorithm to compute this quasi-polynomial function. Our method is based on the multidimensional residue formulae of Szenes-Vergne for computing the number of integral points in polytopes. It provides an answer for a number of low dimensional cases (including the case of 3 quthrits, that is Kronecker coefficients with 3 rows).