Geometry and singularities of the Prony mapping

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Abstract

Prony mapping provides the global solution of the Prony system of equations

$$\Sigma_{i=1}^{n} A_i x_i^k = m_k,\ k = 0, 1, \ldots, 2n-1.$$

This system appears in numerous theoretical and applied problems arising in Signal Reconstruction. The simplest example is the problem of reconstruction of linear combination of δ-functions of the form $g(x) = \sum_{i=1}^{n} a_i \delta(x - x_i)$, with the unknown parameters $a_i,\ x_i,\ i = 1, \ldots, n$, from the "moment measurements" $m_k = \int x^k g(x) dx$.

Global solution of the Prony system, i.e. inversion of the Prony mapping, encounters several types of singularities. One of the most important ones is a collision of some of the points x_i. This singularity is responsible for what is usually called the "super-resolution" problem: solve the Prony system for colliding nodes, in the presence of a strong noise. This is a major difficulty in a number of basic practical problems. In the course of the recent investigation of this type of singularities the role of finite differences was emphasised.

In this talk we describe this and other types of singularities of the Prony mapping, and provide an initial view of its global geometry, which turns out to be fairly complicated. We show, in particular, close connections of the Prony mapping with the "Vieta mapping" expressing the coefficients of a polynomial through its roots, and with hyperbolic polynomials and "Vandermonde mapping" studied by V. Arnold.