STANDING WAVES FOR A CLASS OF KIRCHHOFF TYPE PROBLEMS IN $\mathbb{R}^3$ INVOLVING CRITICAL SOBOLEV EXPONENTS*

Yi He and Gongbao Li[1]

Hubei Key Laboratory of Mathematical Sciences and School of Mathematics and Statistics, Central China Normal University, Wuhan, 430079, P. R. China

ABSTRACT. We are concerned with the following Kirchhoff type equation with critical nonlinearity:

$$\begin{cases} -\Big(\varepsilon^2 a+\varepsilon b\displaystyle\int_{\mathbb{R}^3}|\nabla u|^2\Big)\Delta u+V(x)u=\lambda|u|^{p-2}u+|u|^4u \text{ in } \mathbb{R}^3,\\ u>0,\ u\in H^1(\mathbb{R}^3),\end{cases}$$

where ε is a small positive parameter, $a,b>0$, $\lambda>0$, $2<p\le 4$. Under certain assumptions on the potential V, we construct a family of positive solutions $u_\varepsilon\in H^1(\mathbb{R}^3)$ which concentrates around a local minimum of V as $\varepsilon\to 0$.

Although, critical growth Kirchhoff type problem

$$\begin{cases} -\Big(\varepsilon^2 a+\varepsilon b\displaystyle\int_{\mathbb{R}^3}|\nabla u|^2\Big)\Delta u+V(x)u=f(u)+u^5 \text{ in } \mathbb{R}^3,\\ u>0,\ u\in H^1(\mathbb{R}^3)\end{cases}$$

has been studied in e.g. Y. He, G. Li and S. Peng in [**?**], where the assumption for $f(u)$ is that $f(u)\sim|u|^{p-2}u$ with $4<p<6$ and satisfies the Ambrosetti-Rabinowitz condition which forces the boundedness of any Palais-Smale sequence of the corresponding energy functional of the equation. As $g(u):=\lambda|u|^{p-2}u+|u|^4u$ with $2<p\le 4$ does not satisfy the Ambrosetti-Rabinowitz condition ($\exists\mu>4, 0<\mu\int_0^u g(s)ds\le g(u)u$), the boundedness of Palais-Smale sequence becomes a major difficulty in proving the existence of a positive solution. Also, the fact that the function $\frac{g(s)}{s^3}$ is not increasing for $s>0$ prevents us from using the Nehari manifold directly as usual. Our result extends the main result in Y. He, G. Li and S. Peng in [**?**] concerning the existence and concentration of positive solutions to the case where $f(u)\sim|u|^{p-2}u$ with $4<p<6$.

Key words : existence; concentration; Kirchhoff type equation; critical growth.

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[1]Corresponding Author: Gongbao Li. Email addresses: heyi19870113@163.com (Y. He), lig-b@mail.ccnu.edu.cn(G. Li).