Quantum holography and classical random fields

I.V. Volovich
Steklov Mathematical Institute, Moscow

Workshop on IDAQP and their Applications
(3 - 7 March 2014)

Dedicated to Professor Takeyuki Hida

The Institute for Mathematical Sciences (IMS) at
the National University of Singapore (NUS)

T. Hida. Brownian motion.
Springer (1980)

Т. Хида. Броуновское движение.
Наука (1987)

God does not play dice (Einstein)

<u>God likes to play dice!</u>

Stochastic analysis in mathematical physics

- **Non-Newtonian mechanics**
- **Randomness in classical and quantum mechanics**
- **Time Irreversibility Problem**
- **Measurement problem**
- **Universal boundary value problem for PDE**
- **Quantum holography. AdS/CFT duality**
- **Black Hole Information and Formation Paradoxes**

Newton`s Equation

$$m\frac{d^2}{dt^2}x = F(x),$$

$$x = x(t),$$

$$(M = R^{2n}, \varphi_t)$$

Phase space (q,p), Hamilton dynamical flow

Why Newton`s mechanics can not be true?

- **Newton`s equations of motions use real numbers while one can observe only rationals**
- **Classical uncertainty relations**
- **Time irreversibility problem**
- **Singularities in general relativity**

Classical Uncertainty Relations

Classical Measurements

$$\Delta q > 0, \quad \Delta p > 0$$

$$\Delta t > 0$$

- **Try to solve these problems by developing a new, non-Newtonian mechanics.**

Even for the single particle the fundamental dynamical equation in the proposed "functional" approach is not the Newton equation.

In functional mechanics the basic equation is the Liouville equation or the Fokker - Planck - Kolmogorov type equation. Langevin type eq. for a single particle. Random parameters.

The Newton equation in functional mechanics appears as an approximate equation for the expected values of the position and momentum. Corrections.

States and Observables in Functional Classical Mechanics

$$(q, p) \quad \in \quad \mathbb{R}^2 \qquad \text{(phase space)},$$

$$\rho = \rho(q, p, t) \qquad \text{state of a classical particle}$$

$$\rho \geq 0, \quad \int_{\mathbb{R}^2} \rho(q, p, t) dq dp = 1, \; t \in \mathbb{R} \, .$$

Fundamental Equation in Functional Classical Mechanics

$$\frac{\partial \rho}{\partial t} = -\frac{p}{m}\frac{\partial \rho}{\partial q} + \frac{\partial V(q)}{\partial q}\frac{\partial \rho}{\partial p}\,.$$

Looks like the Liouville equation which is used in statistical physics to describe a gas of particles.

But here we use it to describe a <u>single particle</u>.(moon,…).

<u>Ensemble of observers.</u>

Instead of Newton equation. <u>No trajectories!</u>

<u>All systems are open.</u>

Cauchy Problem

$$\rho\big|_{t=0} = \rho_0(q,p)\,.$$

$$\rho_0(q,p) = \frac{1}{\pi ab} e^{-\frac{(q-q_0)^2}{a^2}} e^{-\frac{(p-p_0)^2}{b^2}}\,.$$

$$a = b = \varepsilon$$

States and Observables in Functional Classical Mechanics

$$\overline{f}(t) = \int f(q,p)\rho(q,p,t)dqdp\,.$$

$f(q,p)$ is a function

Not a generalized function

$$\overline{f}(t) = \langle f \rangle = Ef$$

Corrections to Newton

$$m\frac{d^2}{dt^2}<q>=<F(q)>$$

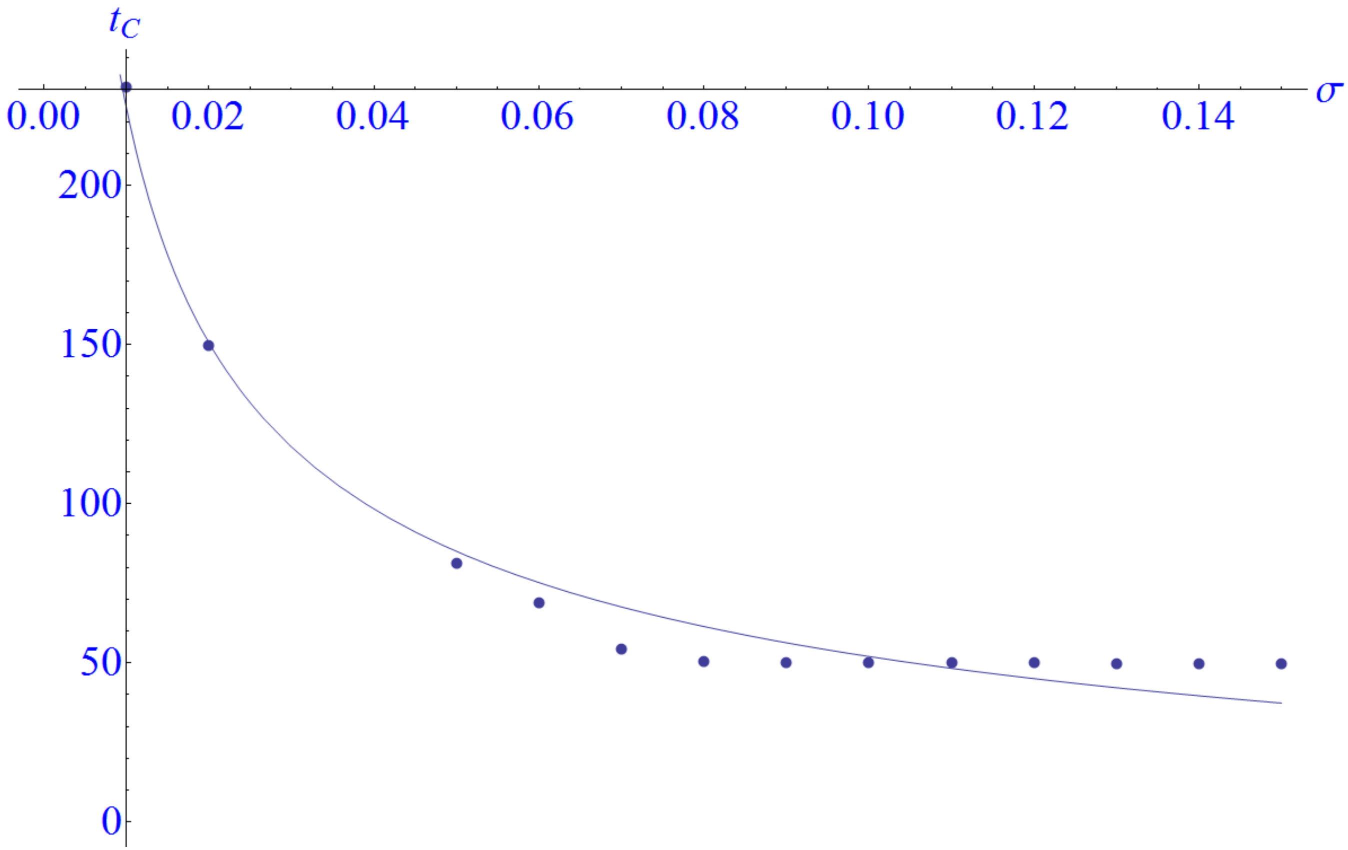
$$\cong F(<q>)+\frac{\varepsilon^2}{2}F''(<q>)$$

Corrections

$$m\frac{d^2}{dt^2} < q(t) > = < F(q)(t) >$$

E. Piskovsky,   A. Mikhailov, O.Groshev,

Problem: h is fixed:
Inoue, Ohya, I.V. Log dependence on time



$$|\langle q\rangle(t_C,\sigma) - q_{KB}(t_C,b)| = 0.1 q_{(KB)}(0,\sigma)$$

$$t_C = O(1/\sqrt{\sigma})$$

- ***I.V. Found. Phys.*, 41:3 (2011), 516.**
- **TMF (2012),…**

- **M. Ohya, I. Volovich,**
- **“Mathematical foundations of quantum information and computation and its applications to nano- and bio-systems”, Springer, 2011.**

Free Motion

$$\frac{\partial \rho}{\partial t} = -\frac{p}{m}\frac{\partial \rho}{\partial q}$$

$$\rho(q, p, t) = \rho_0(q - \frac{p}{m}t, p)\,.$$

Delocalization

$$\rho_c(q,t) = \int \rho(q,p,t)dp = \frac{1}{\sqrt{\pi}\sqrt{a^2 + \frac{b^2t^2}{m^2}}} \exp\{-\frac{(q - q_0 - \frac{p_0}{m}t)^2}{(a^2 + \frac{b^2t^2}{m^2})}\}$$

$$\Delta q^2(t) = \frac{1}{2}(a^2 + \frac{b^2t^2}{m^2})$$

Comparison with Quantum Mechanics

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}$$

$$\rho_q(x,t) = |\psi(x,t)|^2 = \frac{1}{\sqrt{\pi}\sqrt{a^2 + \frac{\hbar^2 t^2}{a^2 m^2}}} \exp\{-\frac{(x - x_0 - \frac{p_0}{m}t)^2}{(a^2 + \frac{\hbar^2 t^2}{a^2 m^2})}\}$$

$$a^2 b^2 = \hbar^2$$

Wigner $$W(q,p,t) = \rho(q,p,t)$$

- **Newton`s approach: Empty space (vacuum) and point particles.**
- **Reductionism: For physics, biology economy, politics (freedom, liberty,…)**

- **This approach: No empty space. Probability distribution. Collective phenomena. Subjective.**

- **T.Hida**

Space-time dependence of entangled states

The FOUNDATION of PHYSICS:

Quantum Field Theory

Quantum Electrodynamics:

- **Quantum Information (Relativistic)**
- **M.Ohya, I.Volovich. “Mathematical Foundations of Quantum Information…”   Springer, 2011**

The FUNDAMENTAL NOTION:

Quantum Field

$$A_j(x) \quad operators$$

$$x = (t, \mathrm{x}) \in R^4$$

QFT LOCALITY

$$[A_j(x), A_i(y)] = 0$$

$$x - y \quad spacelike$$

CORRELATION FUNCTIONS

$$\langle \Psi \,|\, A_j(x) A_i(y) \ldots \,|\, \Psi \rangle$$

PARTICLES

- **Quantum Field (not Particle)**
  is the Basic Notion.

- **Particles are defined in terms of quantum fields:**
  photons, electrons, protons,…

- **A particle is an infinite-dimensional irreducible representation of the Poincare group [m,s]   (for Minkowski space only).**

Quantum nonlocality is considered as an established property of quantum mechanics.

One refers to Bell`s inequalities and experiments with entangled photons.

However, the modern fundamental theory, i.e. quantum field theory, is a local theory.

The notion of locality in quantum field theory is different from the notion of locality in Bell's experiments and in quantum information, where no explicit dependence on spacetime variables is mentioned.

CLASSICAL THEORY IS MORE GENERAL THAN QUANTUM

$$\langle \Psi | A_j B_i | \Psi \rangle =$$

$$\int_{\Lambda} a_j(\lambda)\, b_i(\lambda) dP_{ji}(\lambda)$$

$L.Accardi : Chameleon$

$M.Ohya : Adaptive\ \ dynamics$

$I.V.: Space - time\ dependence$

Bell`s Theorem (A.)

$$\langle \Psi_{spin} | \sigma a \otimes \sigma b | \Psi_{spin} \rangle =$$

$$-ab \neq E\xi(a)\eta(b)$$

Classical random fields $|\xi| \leq 1, |\eta| \leq 1.$

Inappropriate Bell Computation and a Correct Computation for Spacelike Separated Detectors

$$\langle \Psi_{spin} | \sigma a \otimes \sigma b | \Psi_{spin} \rangle = -ab$$

$$\Psi_{spin} = (\Psi_{ij}) \;\; wrong \;\; (J.Bell)$$

$$------------------$$

$$\langle \Psi_{space} | \sigma a \, P_{O1} \otimes \sigma b \, P_{O2} | \Psi_{space} \rangle$$

$$\Psi_{space} = (\Psi_{ij}(\mathrm{x}, \mathrm{y})) \;\; correct \; (nonrelat.) \, (I.V.)$$

SPATIAL DISENTANGLEMENT

$$\langle \Psi_{space} | \sigma a P_{O1} \otimes \sigma b P_{O2} | \Psi_{space} \rangle \to 0$$

$$if \quad dist\ (O1,\ O2\ ) \to 0.$$

Representation Theorem

$$\langle \Psi_{space} | \sigma a P_{O1} \otimes \sigma b P_{O2} | \Psi_{space} \rangle$$

$$= E\xi(a,O1)\eta(b,O2)$$

$$for\ l\mathrm{arg}e\ \ dist\ (O1,\ O2).$$

RADIATION FIELD (PHOTONS) IN WAVEGUIDES

A.Khrennikov, B.Nilsson, T.Ishiwatari, S.Nordebo, I.V.

$$E_j(t,x,y,z) \quad \textit{Maxwell equations}$$

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2} + m_n^{\;2}\right)\varphi_n(t,z) = 0$$

We study the spatial dependence of quantum entangled states and show that the <u>measure of entanglement vanishes for large distances</u>.

<u>Loopholes are unavoidable in Bell experiments</u>.
(Compare the Heisenberg uncertainty relations)

Holography. Gauge/string duality (AdS/CFT)

- The Gauge/Gravity duality gives an correspondence between a gauge theory in the 4-dim physical space and supergravity in the 5-dimensional space where the supergravity is an approximation of the 10-dimensional string theory .

Maldacena '97:

- Or in others words, the properties of the gauge theory in (physical) Minkowski space in 3+1 dimensions are in one-to-one relation with properties of the bulk string (gravity) theory.

- The best known example of such theories is N = 4 super Yang-Mills, a superconformal field theory with matter in the adjoint representation of the gauge group SU(Nc).

Relation between parameters:

$$g^2 = 4\pi g_{\rm st}$$

$$g^2 N_c = \frac{R^4}{\ell_s^4}$$

AdS/CFT correspondence in Euclidean space. T=0

The correspondence between **the strongly coupled N = 4 SYM theory in D=4** and **classical (super)gravity on $AdS_5 x S_5$**

$$ds^2 = \frac{R^2}{z^2}(d\tau^2 + d\mathbf{x}^2 + dz^2) + R^2 d\Omega_5^2$$

Simplest case: $M = \mathrm{AdS}_5 \quad \partial M = \mathrm{R}^4$

$$< \mathcal{O}(\tau_1, \vec{x}_1) ... \mathcal{O}(\tau_n, \vec{x}_n) >, \quad (\tau_k, \vec{x}_k) \in \partial M$$

$$\mathcal{O} \quad \Leftrightarrow \quad \phi$$

$$\phi(\tau, \vec{x}, z), \quad S[\phi], \quad \delta S_{cl}[\phi] = 0$$

$$\text{boundary condition } \phi|_{z=0} = \phi_0$$

$$< e^{\int_{\partial M} \phi_0 \mathcal{O}} > = e^{S_{cl}[\phi_0]}$$

Renormalization $\phi|_{z=\varepsilon} = C(\varepsilon)\, \phi_0$

Witten;Gubser,Polyakov, Klebanov,1998

Arefeva, I.V., PLB, 1998

AdS/CFT correspondence in Euclidean space. T≠0

To compute the Matsubara correlator at finite temperature

$$G^E(k_E) = \int d^4x_E\, e^{-ik_E\cdot x_E} \langle T_E \hat{\mathcal{O}}(x_E)\hat{\mathcal{O}}(0)\rangle$$

Here T_E denotes Euclidean time ordering

$$\left\langle e^{\int_{\partial M}\phi_0\hat{\mathcal{O}}}\right\rangle = e^{-S_{\rm cl}[\phi]_0}$$

boundary condition $\phi|_{z=0} = \phi_0$ **+ requirement of regularity at horizon**

$$ds^2 = \frac{R^2}{z^2}\Big(f(z)d\tau^2 + d\mathbf{x}^2 + \frac{dz^2}{f(z)}\Big) + R^2 d\Omega_5^2$$

$$f(z) = 1 - z^4/z_H^4 \qquad z_H = (\pi T)^{-1}$$

T is the Hawking temperature

Euclidean time coordinate τ is periodic, $\tau \sim \tau + T^{-1}$ $\qquad 0 < z < z_H$

BLACK HOLES in GR. Schwarzschild solution

$$ds^2 = -\left(1 - \frac{r_S}{r}\right) dt^2 + \left(1 - \frac{r_S}{r}\right)^{-1} dr^2 + r^2 d\Omega_2{}^2$$

- **Asymptoticaly flat**
- **Birkhoff's theorem: Schwarzschild solution is the unique spherically symmetric vacuum solution**
- **Singularity**

Information Loss in Black Holes

- **Hawking paradox**. **Pure to mixed states.**
- **Particular case of the Irreversibility problem.**
- **Bogolyubov method of derivation of kinetic equations -- to quantum gravity.**

- Th.M. Nieuwenhuizen, I.V. (2005)

Black hole formation paradox

- One believes that there are many black holes in the Universe
- But there is not enough time after Big Bang to form a black hole.

Gravitational collapse

Oppenheimer – Snyder solution

Uniform perfect sphere of fluid of zero pressure (dust)

The total time of collapse for an observer comoving with the stellar matter is finite, and for this idealized case and typical stellar masses, of the order of a day;

an external observer sees the star asymptotically shrinking to its gravitational radius (infinite time is required to each event horizon).

Fixed classical spacetime?

- **A fixed classical background spacetime does not exists (Kaluza—Klein, Strings, Branes).**

There is a set of classical universes and a probability distribution $\rho(M, g_{\mu\nu})$ which satisfies the Liouville equation (not Wheeler—De Witt).

Stochastic inflation?

Universal boundary value problem for equations of mathematical physics

- V.Zh. Sakbaev, I.V.

A universal boundary value problem for partial differential equations

- **arXiv:1312.4302**

CONCLUSIONS

Stochastic (non-Newtonian) classical mechanics: distribution function instead of individual trajectories.
Newton equation—approximate for average values.
Corrections to Newton`s trajectories.

- **Crucial Role of Spacetime**
- **Quantum Field Theory is a Local Theory**
- **Spatial Disentanglement**

- **Black hole information and formation paradoxes**

THANK YOU!