Stochastic Integration with respect to Gaussian Processes

(*commun work with J.Lévy Vehel*)

Stochastic integration with respect to Gaussian processes, that are not semi martingale, has raised strong interest in recent years motivated especially by its applications in finance and Internet traffic modeling. However, when the Gaussian process considered is not a semi-martingale stochastic integration requires specific developments. If stochastic integration with respect to fractional Brownian motion (fBm), which is the most famous Gaussian process that is not a semi martingale, is well known now it is not the same for any Gaussian process that is not a semi martingale.
The aim of this work is provide a stochastic integral as well as a stochastic calculus with respect to a large class, denoted $\mathcal{C}$, of Gaussian processes $G := (G_t)_{t\in\mathbf{R}}$, assuming G is differentiable in the sense of stochastic distributions (Hida distributions).
After having defined the integral with respect to any gaussian process G of $\mathcal{C}$, which is differentiable in sense of stochastic distributions, we will show how one can build a stochastic calculus (including Itô and Tanaka formulas) with respect to G. The case where G is a Gaussian bridge and a Fractional Brownian motion will then be studied and our stochatic calculus with respect to G will be compared to the existing stochastic calculus (such as the one provided in [1, 6]). In the case where G is a fBm or a multifractional Brownian motion (mBm) we will see that the Wick-Itô integral defined in this work coincide with the one defined in [2, 3] for the fBm and in [5] for the mBm.
Finally, and depending on time, we will present an application of the stochastic calculus with respect to Gaussian processes provided above to the study of local times of Gaussian processes or we will present an application of this stochastic calculus to the study of multifractional volatility models as they have been defined and studied in [4].

Keywords: Gaussian processes, stochastic analysis, fractional Brownian motion, White noise theory, Wick-Itô integral, Pathwise integral.

References

[1] Elisa Alòs, Olivier Mazet, and David Nualart. Stochastic calculus with respect to Gaussian processes. *Ann. Probab.*, 29(2):766–801, 2001.

[2] Elisa Alòs and David Nualart. Stochastic integration with respect to the fractional Brownian motion. *Stoch. Stoch. Rep.*, 75(3):129–152, 2003.

[3] C. Bender. An Itô formula for generalized functionals of a fractional Brownian motion with arbitrary Hurst parameter. *Stochastic Processes and their Applications*, 104:81–106, 2003.

[4] S. Corlay, J. Lebovits, and J. Lévy Véhel. Multifractional stochastic volatility models. *To appear in Mathematical Finance*, 2014.

[5] Joachim Lebovits and Jacques Lévy Véhel. White noise-based stochastic calculus with respect to multi-fractional brownian motion. *To appear in Stochastics*, 2013.

[6] M. Zähle. Integration with respect to fractal functions and stochastic calculus. I. *Probab. Theory Related Fields*, 111(3):333–374, 1998.