Motivational Ratings

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Motivation: Ratings whose goal is to incentivize (moral hazard).

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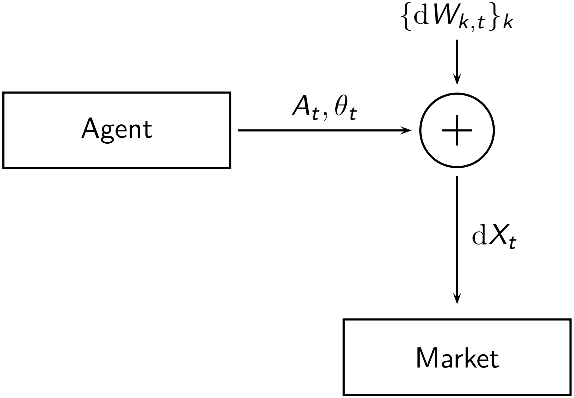
Objective: To understand the structure of optimal ratings.

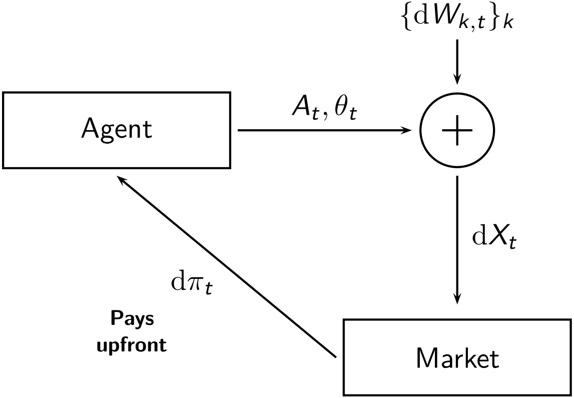
I have no time to compare existing ones.

Unknown skill θ
Private effort A
Forward-looking

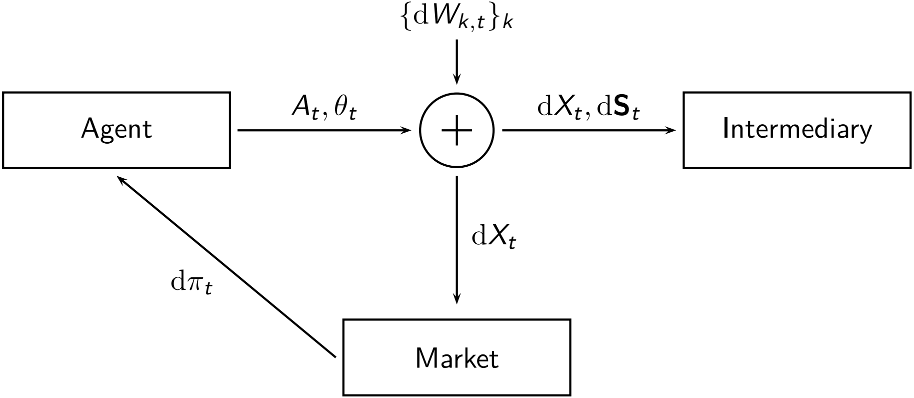
Agent

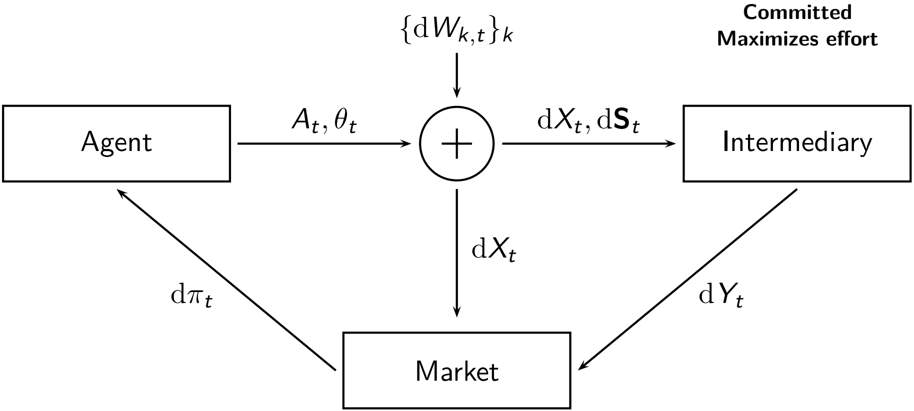


Competitive
Rational expectations



$\{\mathrm{d}W_{k,t}\}_k$
Agent
A_t, θ_t
$+$
$\mathrm{d}X_t$
$\mathrm{d}\pi_t$
Pays
upfront
Market


$\{\mathrm{d}W_{k,t}\}_k$
Agent
A_t, θ_t
+
$\mathrm{d}X_t, \mathrm{d}\mathbf{S}_t$
Intermediary
$\mathrm{d}X_t$
$\mathrm{d}\pi_t$
Market


Committed
Maximizes effort
$\{\mathrm{d}W_{k,t}\}_k$
Agent
A_t, θ_t
$\mathrm{d}X_t, \mathrm{d}\mathbf{S}_t$
Intermediary
$\mathrm{d}X_t$
$\mathrm{d}\pi_t$
$\mathrm{d}Y_t$
Market

Transparency: Career Concerns without Ratings

(a variation on Holmström, 1999)

Continuous time $t \geq 0$, infinite horizon.

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The relevant processes are:

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The relevant processes are:

- Effort: $A_t \in \mathbf{R}_+$; **privately** known by the agent.
- Ability: $\theta_t \in \mathbf{R}$; **unknown** to all.
- Flow output: $\mathrm{d}X_t \in \mathbf{R}$; **commonly** observed (price $=1$).
- Additional information: $\mathrm{d}\mathbf{S}_t \in \mathbf{R}^{K-1}$; **commonly** observed.

Ability Process:

$$\mathrm{d}\theta_t = -\theta_t \mathrm{d}t + \gamma \mathrm{d}W_t^\theta,$$

with $\theta_0 \sim \mathcal{N}(0, \gamma^2/2)$, $\gamma > 0$, and W^θ a standard B.M.

Rate of mean-reversion: 1.

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Rate of mean-reversion: 1.

Output Process:

$$\mathrm{d}X_t = (A_t + \theta_t)\mathrm{d}t + \sigma_1 \mathrm{d}W_{1,t},$$

with $X_0 = 0$, $\sigma_1 > 0$, and W_1 a standard B.M. ($W_1 \perp W^\theta$).

Signal Processes, $k = 2, \ldots, K$:

$$\mathrm{d}S_{k,t} = (\alpha_k A_t + \beta_k \theta_t)\mathrm{d}t + \sigma_k \mathrm{d}W_{k,t},$$

with $S_{k,0} = 0$, $\sigma_k > 0$, $\alpha_k, \beta_k \in \mathbb{R}$ and W_k a standard B.M.

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We also write $S_1 \coloneqq X$.

Learning

Let

$$\mathcal{F}_t := \sigma(\{X_s, \mathbf{S}_s\}_{s \leq t})$$

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An asterisk (*e.g.*, $\mathbf{E}^*$) refers to the law of θ under expected effort.

Operators without it (*e.g.*, $\mathbf{E}$) refer to the law under true effort.

Given a conjectured (deterministic) A^*, the mean belief follows

$$\mu_t = \mathbf{E}^*[\theta_t \mid \mathcal{F}_t] = \int_{s \le t} e^{-\kappa(t-s)} \sum_{k=1}^{K} \frac{\beta_k}{\sigma_k^2} \left(dS_{k,s} - \alpha_k A_s^* ds\right),$$

where

$$\kappa := \sqrt{1 + \gamma^2 \sum_{k=1}^{K} \frac{\beta_k^2}{\sigma_k^2}}.$$

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$$\mu_t = \mathbf{E}^*[\theta_t \mid \mathcal{F}_t] = \int_{s \le t} \underbrace{e^{-\kappa(t-s)}}_{\text{Decay}} \sum_{k=1}^{K} \overbrace{\frac{\beta_k}{\sigma_k^2}}^{\text{Signal to noise ratio}} \underbrace{(dS_{k,s} - \alpha_k A_s^* ds)}_{\text{Innovation}},$$

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The belief depreciates at rate κ.

Payoffs

Given a (cumulative) transfer process π, realized payoffs are:

$$\begin{aligned} \text{Market:} \quad & \int_0^\infty e^{-rt}(\ dX_t \quad - \quad d\pi_t\ ), \\ \text{Agent:} \quad & \int_0^\infty e^{-rt}(\ d\pi_t \quad - \quad c(A_t)dt\ ), \end{aligned}$$

where the discount rate is $r > 0$, and $c(0) = c'(0) = 0$ and $c'' > 0$.

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where the discount rate is $r > 0$, and $c(0) = c'(0) = 0$ and $c'' > 0$.

Recall that $\mathbf{E}[dX_t] = A_t dt$.

Hence, **efficiency** requires $c'(A_t) = 1\ \forall t$.

An <u>equilibrium</u> is $(\mathcal{A}, \Pi)$, with $A_t = \mathcal{A}(t)$, $\pi_t = \Pi(t, \mathcal{F}_t)$, s.t.

1. (0-profit)

$$\pi_\tau = \int_0^\tau \mathbf{E}^*[A_t^* + \theta_t \mid \mathcal{F}_t]\mathrm{d}t, \qquad \forall \tau.$$

2. (Optimal effort)

$$A \in \arg\max_{\widetilde{A}} \mathbf{E}\left[\int_0^\infty e^{-rt}(\mathrm{d}\pi_t - c(\widetilde{A}_t)\mathrm{d}t)\right].$$

3. (Correct beliefs)

$$A_t = A_t^*, \qquad \forall t.$$

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Here is what drives incentives.

Here is what drives incentives. It's simple.

$$
\begin{tikzcd}
{} \arrow[r] & \mu_t \arrow[dr] & \\
& & \mathrm{d}\pi_t \\
& A_t^* \arrow[ur] &
\end{tikzcd}
$$

$$
\begin{tikzcd}
{} \arrow[r] & \mu_t \arrow[dr] & & \\
& & \mathrm{d}\pi_t & \\
& A_t^* \arrow[ur] & A_t \arrow[dr] & \\
& & & \mathrm{d}X_t \\
{} \arrow[rr] & & \theta_t \arrow[ur] &
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\begin{tikzcd}
{} \arrow[r] & \mu_t \arrow[drr] & & & \\
& & & \mathrm{d}\pi_t & \\
& A_t^* \arrow[urr] & = & A_t \arrow[dr] & \\
& & & & \mathrm{d}X_t \\
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\begin{tikzcd}
{} \arrow[r] & \mu_t \arrow[drr] & & & \\
& & & \mathrm{d}\pi_t & \\
& A_t^* \arrow[urr] & = & A_t \arrow[dr, "\mathrm{d}t"] & \\
& & & \mathrm{d}W_t \arrow[r] & \mathrm{d}X_t \\
{} \arrow[rrr] & & & \theta_t \arrow[ur, "\mathrm{d}t"'] &
\end{tikzcd}
$$

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\begin{tikzcd}
{} \arrow[r] & \mu_t \arrow[rrrr] \arrow[drr] & & & & \mu_{t+\mathrm{d}t} \\
& & & \mathrm{d}\pi_t & & \\
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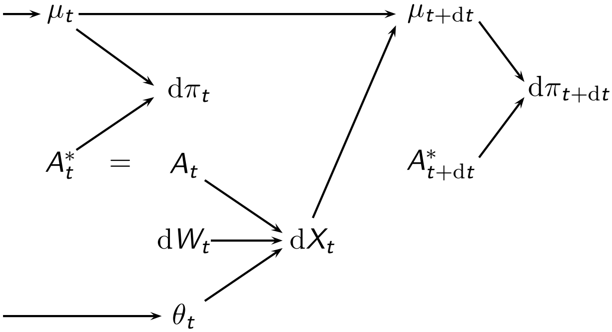
$$



μ_t
$\mu_{t+\mathrm{d}t}$
$\mathrm{d}\pi_t$
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$A_t^* = A_t$
$A_{t+\mathrm{d}t}^*$
$\mathrm{d}W_t$
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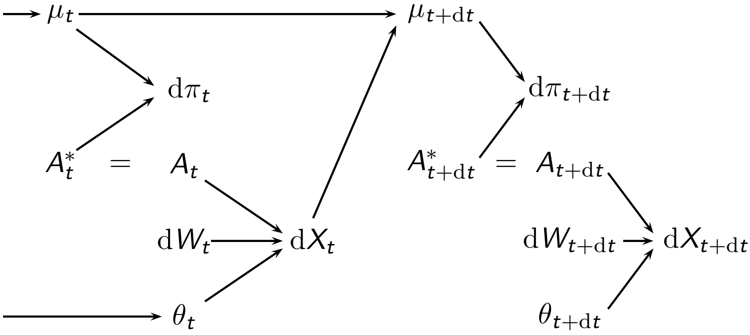
θ_t



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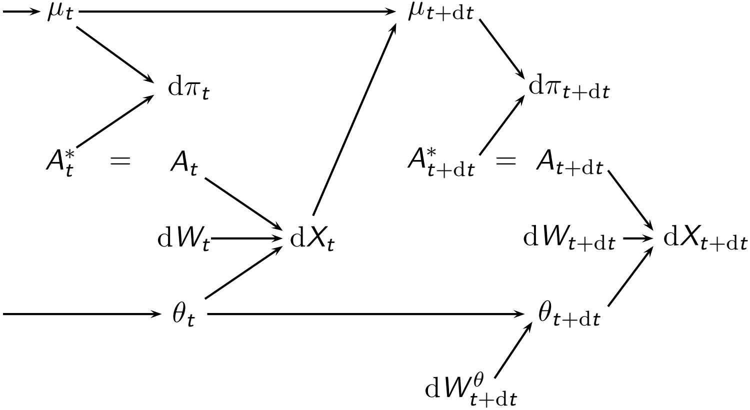
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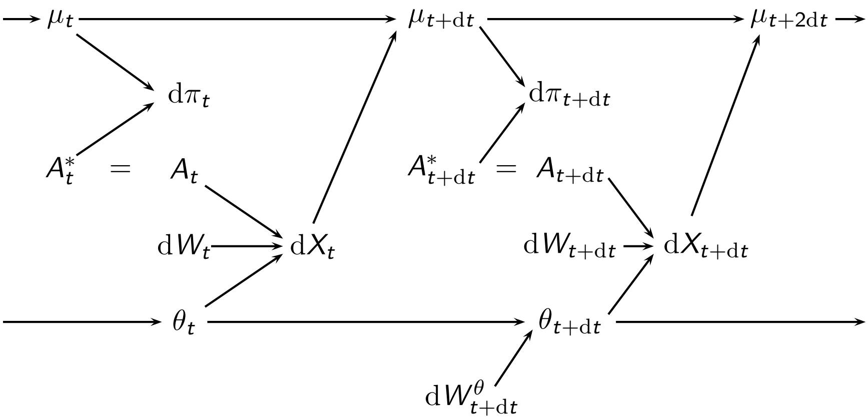
θ_t
$\theta_{t+\mathrm{d}t}$
$\mathrm{d}W_{t+\mathrm{d}t}^{\theta}$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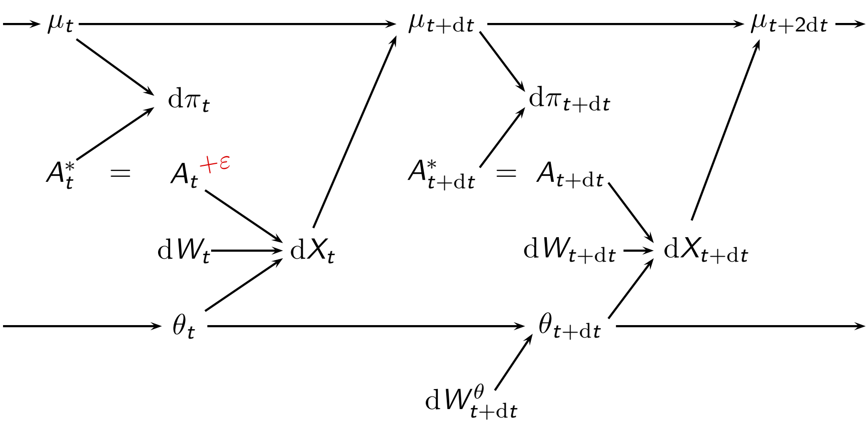
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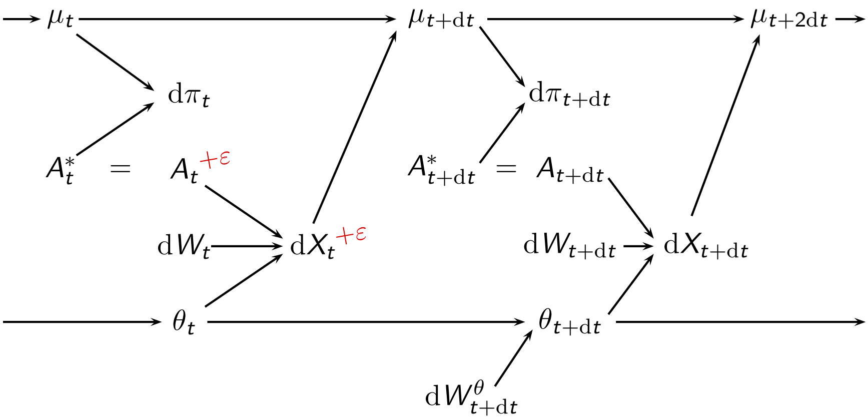
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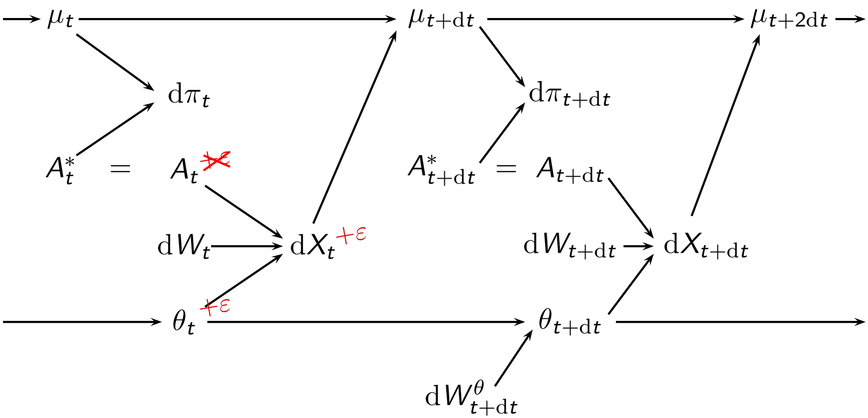

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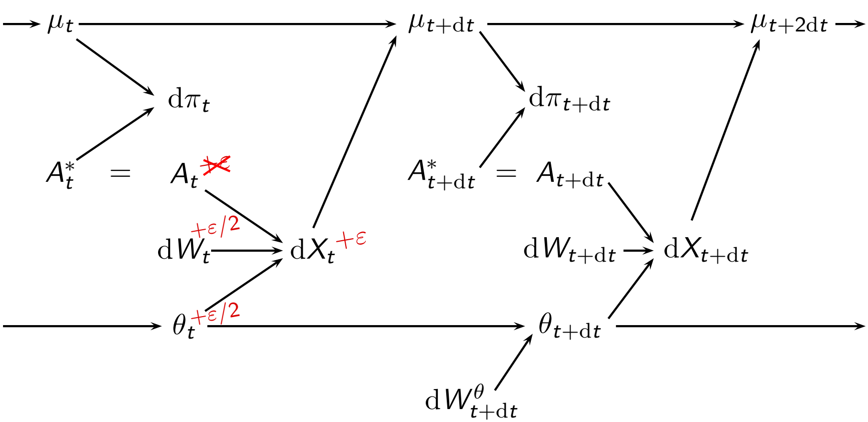

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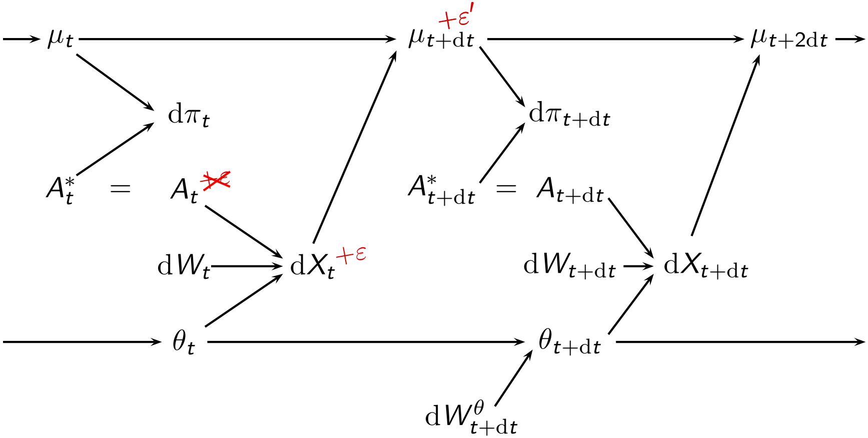
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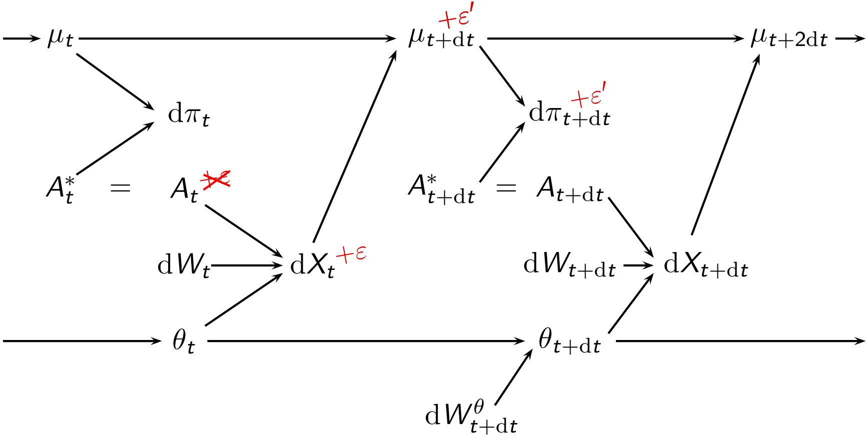
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θ_t
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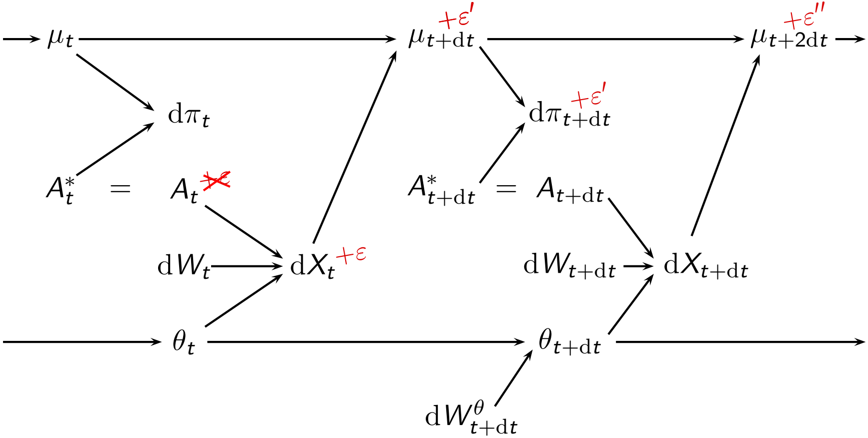

μ_t
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$\mu_{t+2\mathrm{d}t}^{+\varepsilon''}$
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θ_t
$\theta_{t+\mathrm{d}t}$
$\mathrm{d}W_{t+\mathrm{d}t}^{\theta}$

Hence, the agent maximizes over $\mathcal{A}$:

$$\mathbf{E}\left[\int_0^\infty e^{-rt}(\mu_t - c(A_t))\mathrm{d}t\right],$$

where $\mu_t = \mathbf{E}^*[\theta_t \mid \mathcal{F}_t]$ is computed given the optimal A^*.

Full Transparency

Focus on steady state.

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What is?

Rating Systems

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So she wants to "pick $\mathcal{F}$" so that the argmax is largest.

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Given time, will focus on **exclusive** systems.

Rating Processes

A rating process is defined as a vector-valued process $\mathbf{Y}$ s.t.

1. For all t, $\mathbf{Y}_t$ is measurable wrt. $\mathcal{G}_t$.
2. For all Δ, $(\mathbf{Y}_t, \mathbf{S}_t - \mathbf{S}_{t-\Delta})$ is normal and stationary.
3. The map $\Delta \mapsto \mathbf{Cov}[\mathbf{Y}_t, \mathbf{S}_{t-\Delta}]$ is piecewise C^1.
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Methods that Qualify:

Exponential smoothing. (Business Week's b-school ranking.)

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Methods that Don't:

Coarse ratings.

Exclusion from the rating system after underperformance.

What can the Intermediary Do? More Generally:

$$
\begin{array}{cccccc}
\cdots & \mathrm{d}X_s & \cdots & \mathrm{d}X_{t-\mathrm{d}t} & \mathrm{d}X_t \\
 & \vdots & & \vdots & \vdots \\
\cdots & \mathrm{d}S_{k,s} & \cdots & \mathrm{d}S_{k,t-\mathrm{d}t} & \mathrm{d}S_{k,t} \\
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$$
\begin{array}{cccccc}
 & e^{-\delta(t-s)} & + & e^{-\delta \mathrm{d}t} & + & 1 \\
\cdots & \mathrm{d}X_s & \cdots & \mathrm{d}X_{t-\mathrm{d}t} & & \mathrm{d}X_t \\
 & \vdots & & \vdots & & \vdots \\
\cdots & \mathrm{d}S_{k,s} & \cdots & \mathrm{d}S_{k,t-\mathrm{d}t} & & \mathrm{d}S_{k,t} \\
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 & \vdots & & \vdots & \vdots & & \\
\cdots & \mathrm{d}S_{k,s} & \cdots & \mathrm{d}S_{k,t-\mathrm{d}t} & \mathrm{d}S_{k,t} & & \\
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Lemma.

Fix a rating process $\mathbf{Y}$*. Given a conjectured effort level* A^**, there exist vector-valued functions* $\mathbf{u}_k$*,* $k = 1, \ldots, K$*, such that, for all* t*,*

$$\mathbf{Y}_t = \sum_{k=1}^{K} \int_{-\infty}^{t} \mathbf{u}_k(t-s)(\mathrm{d}S_{k,s} - \alpha_k A^* \mathrm{d}s).$$

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The Optimal Rating System

The unique optimal **confidential** rating system is

$$u_k(t) = d_k \frac{\sqrt{r}}{\lambda} e^{-rt} + \frac{\beta_k}{\sigma_k^2} e^{-\kappa t}.$$

Here,

$$d_k := (\kappa^2 - r^2) m_\beta \frac{\alpha_k}{\sigma_k^2} - (\kappa^2 - 1) m_{\alpha\beta} \frac{\beta_k}{\sigma_k^2},$$

with

$$\begin{aligned} \lambda &:= (\kappa - 1)\sqrt{r}(1 + r) m_{\alpha\beta} + (\kappa - r)\sqrt{\Delta}, \\ \Delta &:= (\kappa + r)^2 (m_\alpha m_\beta - m_{\alpha\beta}^2) + (1 + r)^2 m_{\alpha\beta}^2, \quad m_\alpha := \sum_{k=1}^{K} \frac{\alpha_k^2}{\sigma_k^2}. \end{aligned}$$

That is,

$$Y_t = \int_{-\infty}^{t} \sum_{k=1}^{K} \Big(d_k \frac{\sqrt{r}}{\lambda} e^{-r(t-s)} + \frac{\beta_k}{\sigma_k^2} e^{-\kappa(t-s)} \Big)(\mathrm{d}S_{k,s} - \alpha_k A^* \mathrm{d}s).$$

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One state is the rating Y_t.

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Laws of motion:

$$\begin{aligned}
\mathrm{d}\nu_t =& -\kappa\nu_t \mathrm{d}t + \frac{\gamma^2}{\kappa+1}\sum_k \frac{\beta_k}{\sigma_k^2}\left(\mathrm{d}S_{k,t} - \alpha_k A^* \mathrm{d}t\right), \\
\mathrm{d}Y_t =& -\left[rY_t - \frac{(\kappa+1)(r-\kappa)}{\gamma^2}\nu_t\right]\mathrm{d}t \\
&+ \frac{\sqrt{r}}{\lambda}\sum_k \left(d_k + \frac{\beta_k}{\sigma_k^2}\right)\left(\mathrm{d}S_{k,t} - \alpha_k A^* \mathrm{d}t\right).
\end{aligned}$$

Two states are needed: keeping track of ν_t isn't enough.

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The intermediary's belief isn't a summary statistic for the rating given $\{\mathbf{S}_s\}_{s \leq t}$. Neither is the "last" rating, given the innovation.

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The rating process Y isn't Markov. The pair (ν, Y) is.

Reality Check

Ratings are not Markov: widely documented for credit rating.

Altman and Kao (1992), Carty and Fons (1993), Altman (1998), Nickell et al. (2000), Bangia et al. (2002), Lando and Skødeberg (2002), Hamilton and Cantor (2004), etc.

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Mixture rating models: shown to explain economic differences.

Two-state: Frydman and Schuerman (2008);
HMM: Giampieri et al. (2005);
Rating momentum: Stefanescu et al. (2006).

Implication: Benchmarking

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$$\alpha_k = \alpha, \beta_k = \beta, \sigma_k = \sigma.$$

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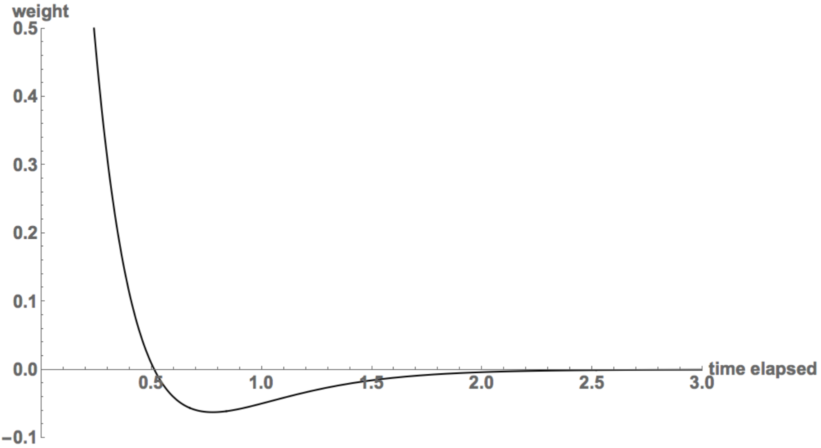
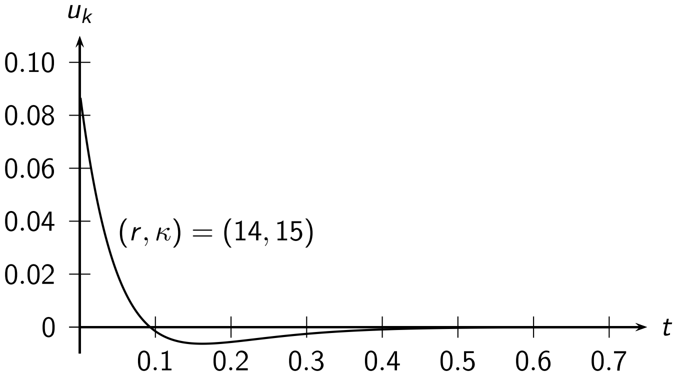
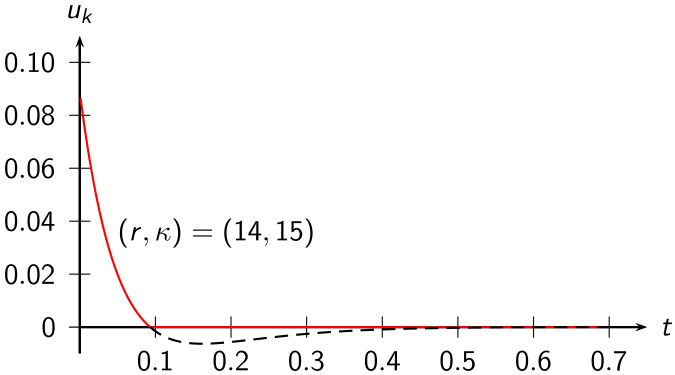
So the incentive state isn't always added. It may be subtracted.

This is the common weight $u_k(t)$ given to past signals.




u_k
0.10
0.08
0.06
0.04
0.02
0
$(r, \kappa) = (14, 15)$
0.1
0.2
0.3
0.4
0.5
0.6
0.7
t



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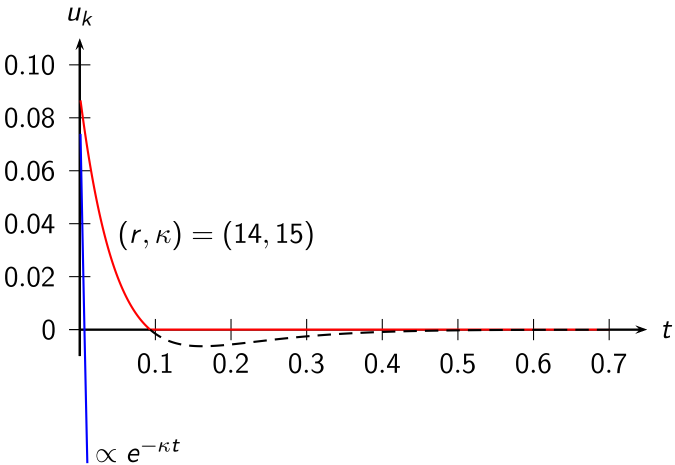
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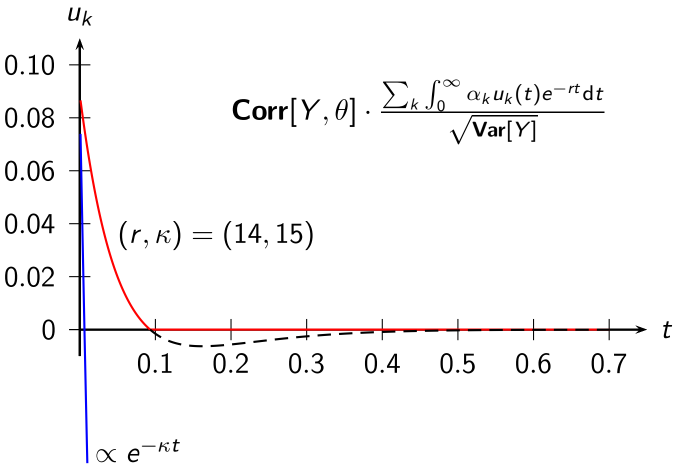
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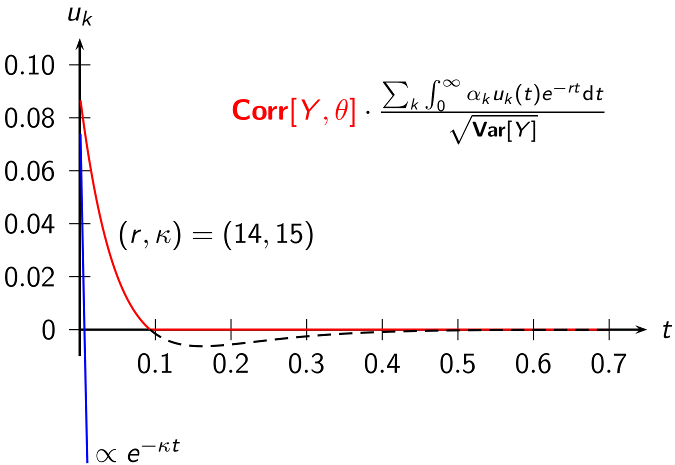

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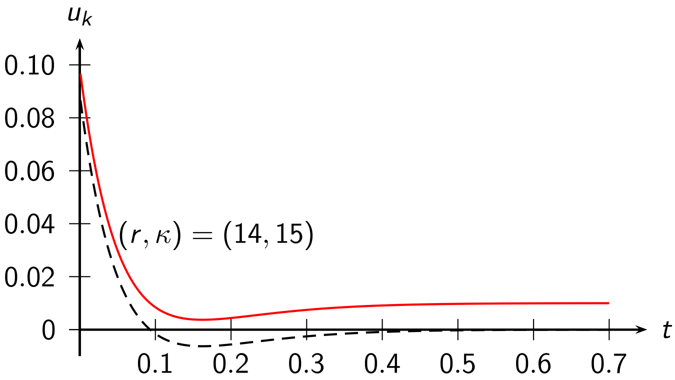
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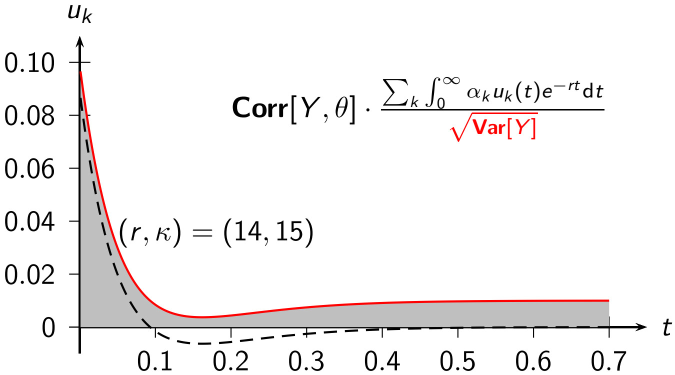

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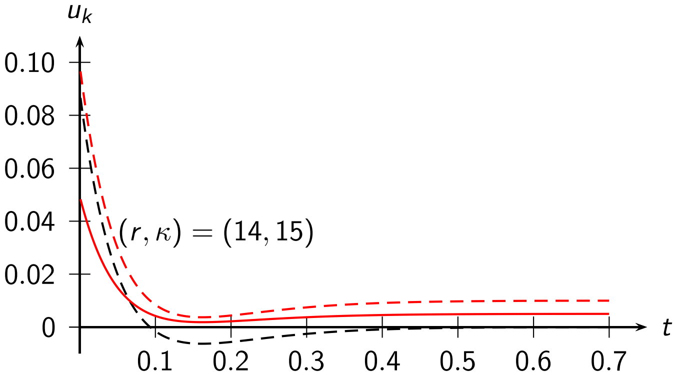

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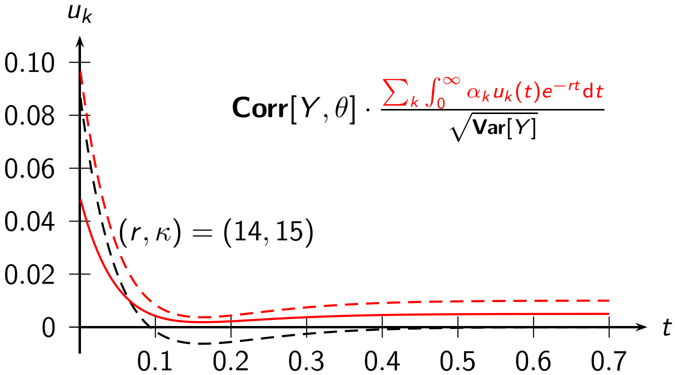
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Reality Check

Benchmarking: Prior-year performance widely used for incentives.

> *When standards are based on prior-year performance, managers might avoid unusually positive performance outcomes, since good current performance is penalized in the next period through an increased standard.* —Murphy, 2001.

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Lemma.
A rating process Y is proportional to a public rating system belief iff

$$\mathbf{Corr}[Y_{t+\Delta}, Y_t] = \mathbf{Corr}[\theta_{t+\Delta}, \theta_t] \textit{ for all } \Delta \geq 0.$$

Public Ratings

The unique optimal **public** rating system is

$$u_k(t) = \widetilde{d}_k \frac{\sqrt{r}}{\lambda} e^{-\sqrt{r}t} + \frac{\beta_k}{\sigma_k^2} e^{-\kappa t}.$$

Here,

$$\widetilde{d}_k := \frac{\kappa - \sqrt{r}}{\kappa - r} d_k + \lambda \frac{\sqrt{r} - 1}{\kappa - r} \frac{\beta_k}{\sigma_k^2}.$$

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In common:	Differences:
A two-state rating system.	Impulse response is the harmonic mean between the discount rate and the rate of mean-reversion.
One state is the belief.	**With homogeneous signals, $\tilde{d}_k = 0$: transparency is best.**
No signal gets discarded.	
Benchmarking can arise.	

Some of the Technical Difficulties

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Here, each u_k is function of a single variable (time), so

$$\int_a^b \int_c^d H(x, y, u(x), u(y), u'(x), u'(y)) \mathrm{d}x \mathrm{d}y,$$

or more precisely, a time-delayed problem such as

$$\int_a^b \int_c^d \int_e^f H(x, y, u(x), u(y+t), u'(x), u'(y+t)) \mathrm{d}x \mathrm{d}y \mathrm{d}t.$$

In addition, in the public case, we have a continuum of constraints:

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We guess some $\lambda(\Delta)$ and replace the constraints with (roughly)

$$\int_{\Delta \geq 0} \lambda(\Delta) \left(\mathbf{Corr}[Y_{t+\Delta}, Y_t] - e^{-\Delta} \right) \mathrm{d}\Delta = 0,$$

and solve the isoperimetric problem.

We check *ex post* that all constraints are satisfied.

Sufficiency

Hence, we have a minimization problem of the type

$$L(\mathbf{u}) := \mathbf{u} \mapsto F(\mathbf{u}) + \lambda G(\mathbf{u}).$$

Standard sufficiency theorems (*e.g.*, fields of extremals) don't apply.

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It's a bit more complicated here, but same idea. We guess a constant H s.t. $L + H$ is "nice," *e.g.*, in the scalar case, for all u,

$$L(u) + H = \int_0^\infty h(y) \left(\int_0^\infty k(y,t) u(t) \mathrm{d}t \right)^2 \mathrm{d}y.$$

for some $h : \mathbb{R}_+ \to \mathbb{R}_+$,$k : \mathbb{R}_+^2 \to \mathbb{R}$ such that, for all y,

$$\int_0^\infty k(y,t) u(t) \mathrm{d}t = 0 \Rightarrow u \propto u^*.$$

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Exclusive vs. Non-Exclusive Information

Suppose some (not all) signals are openly available to the market.

In common:

New features:
(With homogeneous signals)

A two-state rating system.

Private:

$$u_k = \hat{d}_k e^{-rt} + \frac{\beta_k}{\sigma_k^2} e^{-\kappa t}$$

Public:

$$u_k = \check{d}_k e^{-\delta t} + \frac{\beta_k}{\sigma_k^2} e^{-\kappa t}$$

Better informed market.

Public information and ratings can be substitutes.

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Probably not. But they illustrate possibilities:

> Insisting on transparency or even publicness isn't optimal.

And, more surprisingly:

> Markovian rating systems aren't either.
>
> Benchmarking can be.

How do Different Signals get Weighted?

The confidential process can be rewritten as

$$u_k(t) = \frac{\beta_k}{\sigma_k^2}\left[\left((\kappa^2 - r^2)\frac{\alpha_k}{\beta_k} - (\kappa^2 - 1)\frac{m_{\alpha\beta}}{m_\beta}\right)\frac{\sqrt{r}\, m_\beta}{\lambda} e^{-rt} + e^{-\kappa t}\right].$$

Fixing the SNR $\frac{\beta_k}{\sigma_k^2}$, signals are ordered according to the ratio $\frac{\alpha_k}{\beta_k}$: the higher the ratio, the larger the weight (whether positive or not).

Why Two States, Why the Impulse Response r?

Consider the following example with $K = 2$:

$$\beta = \beta_1 > 0, \alpha_1 = 0, \quad \alpha = \alpha_2 > 0, \beta_2 = 0,$$

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Computing the marginal cost, given (c, δ),

$$c'(A^*) = \quad \frac{\sqrt{\delta}}{r + \delta} \qquad \frac{c}{1 + c^2} \qquad \frac{2\alpha\beta}{(1 + \kappa)\sigma^2}.$$

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The analysis extends to multi-dimensional actions (separable cost).

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As an example:

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with cost $c(a_1, a_2) = c \cdot (a_1^2 + a_2^2)$. The best confidential system is

$$u_1(t) = \frac{\sqrt{r}}{\sigma_1} e^{-rt}, \quad u_2(t) = \frac{e^{-\kappa t}}{\sigma_2^2},$$

and effort

$$c'(a_1) = \frac{\kappa - 1}{4\sqrt{r}\sigma_1}, \quad c'(a_2) = \frac{\kappa - 1}{2(r + \kappa)\sigma_2^2}.$$

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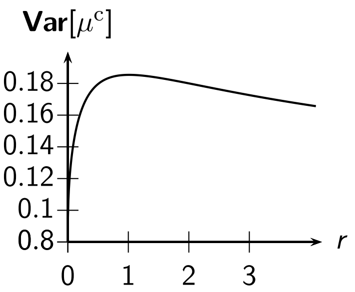
Variance non-monotone in r.



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When mean-reversion tends to 0:

Effort converges to a finite limit; no transparency.